Throughput and Heavy-traffic Optimality of General Load Balancing Algorithm in Cloud Networks

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Outline

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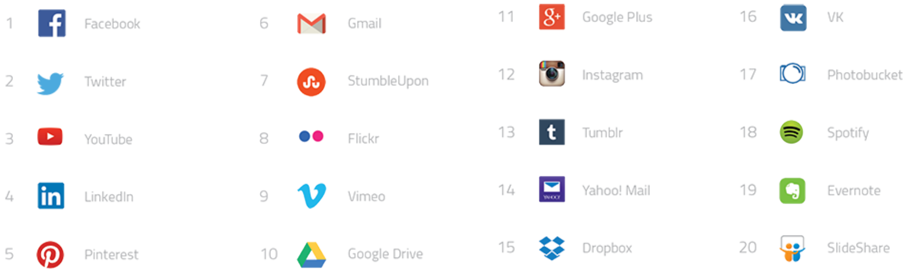
1. Introduction and Motivation
   - Why do we need an effective and fast cloud?
   - Load Balancing and Previous Works
2. Throughput and Heavy-traffic optimality of General Load Balancing
   - Model, Challenges, Contributions and Key insights
   - Methodology and Sufficient Conditions
   - Homogeneous Servers
   - Heterogeneous Servers
   - Conclusions

Cloud is everywhere in life



Source: *https://www.skyhighnetworks.com/*

Figure: Top 20 consumer cloud services in 2016

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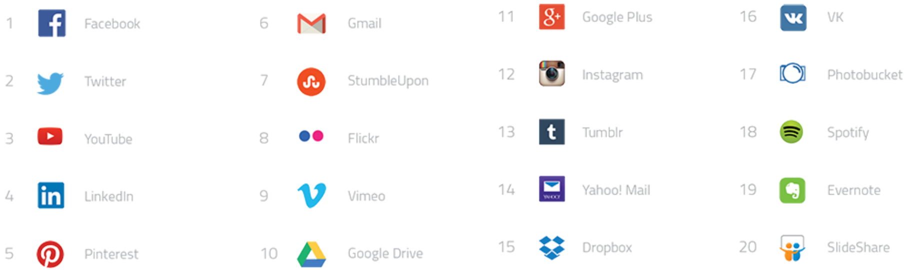

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Question: What makes a good cloud service?

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Question: How do we design an effective and fast cloud system?

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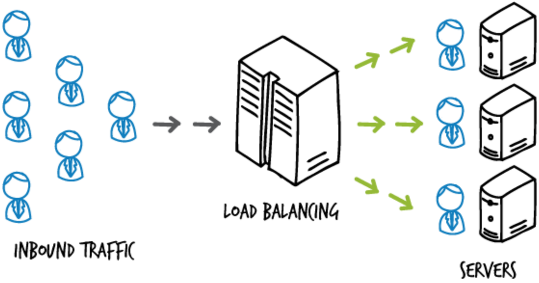

Figure: A typical model in cloud system

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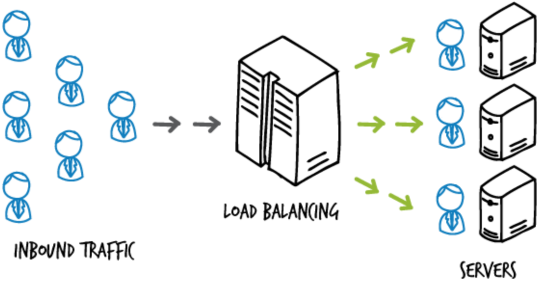

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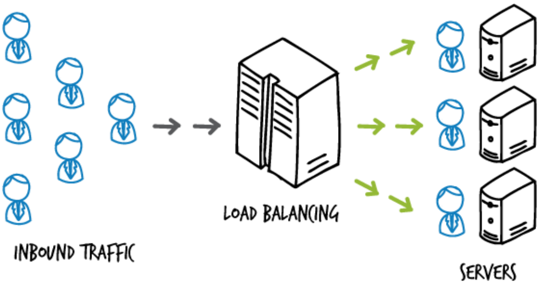

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- **Load balancing:** Choose the right server(s) when requests coming.
  - It is the key to optimize resource use, maximize throughput, reduce response time in cloud system.
  - It becomes more and more critical due to explosive increase in the number of servers and traffic in cloud system.

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Question: Do we really need sampling for each arrival?

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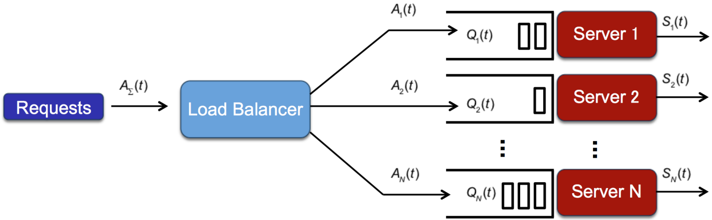

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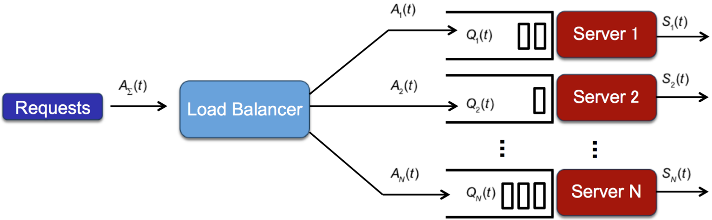

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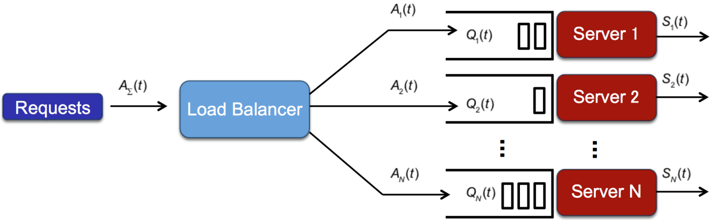

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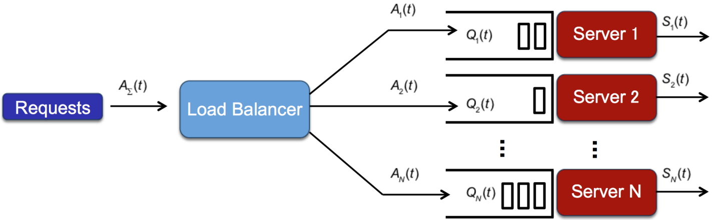

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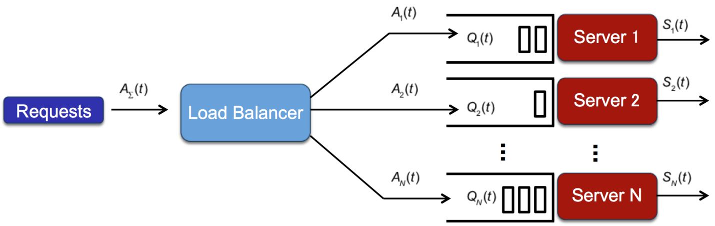

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- Requests cannot be removed from server to server.
- Queue length dynamic is

$$
\begin{aligned}
Q_n(t+1) &= [Q_n(t) + A_n(t) - S_n(t)]^+ \\
&= Q_n(t) + A_n(t) - S_n(t) + U_n(t).
\end{aligned}
\tag{1}
$$

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Definition (Throughput Optimality)

In general, a load balancing policy is said to be throughput optimal if it can stabilize any set of arrival rates which can be stabilized by another policy. In our model, a load balancing policy is throughput optimal if it can stabilize any exogenous rate $\lambda_\Sigma < \mu_\Sigma$.

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Definition (Heavy-traffic Optimality)

A load balancing policy is said to be heavy-traffic optimal if the expected steady-state sum queue length is asymptoticly the same as the equivalent single queue when the arrival rate approaches to the capacity boundary.

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  - The 'good' policy can be any one time-slot sampling heavy traffic optimal policy, such as JBA, Power-of-d, and JSQ depending on different situations.

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- As the traffic gets heavier, it actually becomes more and more easy to achieve heavy-traffic optimality.
  - The upper bound on the sampling interval T gets larger when the traffic becomes heavier.
  - In some sense, load balancing becomes easier (kind of counterintuitive, but can be explained) when traffic becomes heavier.

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Main Ideas

- Choose the Lyapunov function $Z(\mathbf{Q}) \triangleq \|\mathbf{Q}\|_1^2 = \left(\sum_{n=1}^{N} Q_n\right)^2$, and set the mean drift to zero in steady-state to get bounds.

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- Assume (R1) and (R2) satisfy, we obtain the expected sum queue length in steady-state

$$\epsilon\mathbb{E}\left[\sum_{n=1}^{N} \overline{Q}_n^{(\epsilon)}\right] = \frac{\zeta^{(\epsilon)}}{2} + \mathbb{E}\left[\left\|\overline{\mathbf{Q}}(t+1)\right\|_1 \left\|\overline{\mathbf{U}}(t)\right\|_1\right] - \mathbb{E}\left[\left\|\overline{\mathbf{U}}(t)\right\|_1^2\right] \quad (2)$$

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- By letting $N = 1$, expected queue length for the corresponding Equivalent Single Queue is simply

$$\epsilon\mathbb{E}\left[q^{(\epsilon)}\right] = \frac{\zeta^{(\epsilon)}}{2} + \mathbb{E}\left[q(t+1)u(t)\right] - \mathbb{E}\left[u^2\right] \quad (3)$$

Main Ideas (Cont'd)

- Foe the equivalent single queue, by exploiting the most important equation $q(t+1)u(t) = 0$ and the fact $\mathbb{E}\left[u^2\right]$ is $o(\epsilon)$, we have

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- Foe the N server, by exploiting the equation $Q_n(t+1)U_n(t) = 0$ and the fact $\mathbb{E}\left[\left\|\overline{\mathbf{U}}(t)\right\|_1^2\right]$ is $o(\epsilon)$, we have

$$\begin{aligned}\epsilon\mathbb{E}\left[\sum_{n=1}^{N}\overline{Q}_n^{(\epsilon)}\right] &= \frac{\zeta^{(\epsilon)}}{2} + \mathbb{E}\left[\left\|\overline{\mathbf{Q}}(t+1)\right\|_1\left\|\overline{\mathbf{U}}(t)\right\|_1\right] - \mathbb{E}\left[\left\|\overline{\mathbf{U}}(t)\right\|_1^2\right] \\ &= \frac{\zeta^{(\epsilon)}}{2} + N\mathbb{E}\left[\langle\overline{\mathbf{U}}, -\overline{\mathbf{Q}}_\perp(t+1)\rangle\right] - o(\epsilon) \\ &\leq \frac{\zeta^{(\epsilon)}}{2} + \sqrt{\mathbb{E}\left[\left\|\overline{\mathbf{Q}}_\perp\right\|^2\right]o(\epsilon)} - o(\epsilon)\end{aligned}$$

in which $\mathbf{Q}_\perp$ is the perpendicular component of $\mathbf{Q}$ with respect to $\mathbf{c} = \frac{1}{\sqrt{N}}\mathbf{1}$

Sufficient Conditions for Throughput Heavy-traffic Optimal

- (R1) If $\overline{\mathbf{Q}}$ exists, i.e., the underline Markov chain is positive recurrent.
- (R2) If $\mathbb{E}\left[\left\|\overline{\mathbf{Q}}\right\|^2\right] \le M$, i.e., the second moments is bounded by a constant.
- (R3) If $\mathbb{E}\left[\left\|\overline{\mathbf{Q}}_\perp\right\|^2\right] \le K$, which is independent of ϵ. This is often called steady-state collapse, which indicates that under heavy-traffic, queue-length vector concentrates around the line $\mathbf{c}$.

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How can we bound the moments?

A Very Useful Lemma

Lemma

For an irreducible aperiodic and positive Markov chain $\{X(t), t \geq 0\}$ *over a countable state space* $\mathcal{X}$*, which converges in distribution to* $\overline{X}$*, and suppose* $V : \mathcal{X} \to \mathbb{R}_+$ *is a Lyapunov function. We define the* T *time slot drift of* V *at* X *as*

$$\Delta V(X) := [V(X(t_0 + T)) - V(X(t_0))]\mathcal{I}(X(t_0) = X),$$

where $\mathcal{I}(.)$ *is the indicator function. Suppose for some positive finite integer* T*, the* T *time slot drift of* V *satisfies the following conditions:*

- *(C1) There exists an* $\eta > 0$ *and a* $\gamma < \infty$ *such that for any* $t_0 = 1, 2, \ldots$ *and for all* $X \in \mathcal{X}$ *with* $V(X) \geq \gamma$*,*

$$\mathbb{E}\left[\Delta V(X) | X(t_0) = X\right] \leq -\eta.$$

- *(C2) There exists a constant* $D < \infty$ *such that for all* $X \in \mathcal{X}$*,*

$$\mathbb{P}(|\Delta V(X)| \leq D) = 1.$$

Then, there exist finite constants $\{M_r, r \in \mathbb{N}\}$ *such that for each positive* r*,* $\mathbb{E}\left[V(\overline{X})^r\right] \leq M_r$*, where* M_r *are fully determined by* η*,* γ *and* D*.*

General Sufficient Conditions

Assuming bounded support of arrival and departure process, by exploiting the useful lemma and properties of projection to a convex cone, we are able to give sufficient condition to more general cases

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- (S1) If there exists a finite constant T_1 and $K_1 > 0$ and $\delta > 0$ such that for all t_0

$$\mathbb{E}\left[\sum_{t=t_0}^{t_0+T_1-1} \langle \mathbf{Q}(t), \mathbf{A}(t) - \mathbf{S}(t) \rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \leq -\delta \|\mathbf{Q}\| + K_1 \tag{4}$$

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- (S2) If there exists a finite constant T_2, and constants $K_2 > 0$ and $\eta > 0$, both independent of ϵ such that for all t_0

$$\mathbb{E}\left[\sum_{t=t_0}^{t_0+T_2-1} \langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t) \rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \leq -\eta \|\mathbf{Q}_\perp\| + K_2 \tag{5}$$

  holds, then the moments of perpendicular component with respect to any convex set $\mathcal{C}$ is bounded. (steady-state collapse to a convex set $\mathcal{C}$)

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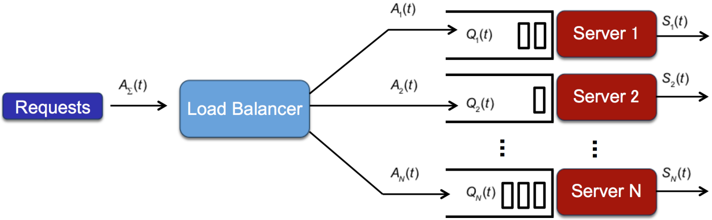
Clearly, if the convex set is $\mathbf{c} = \frac{1}{\sqrt{N}}\mathbf{1}$, (S1) implies (R1-R2) and (S2) implies (R3).

Outline

JBA in Homogeneous Servers



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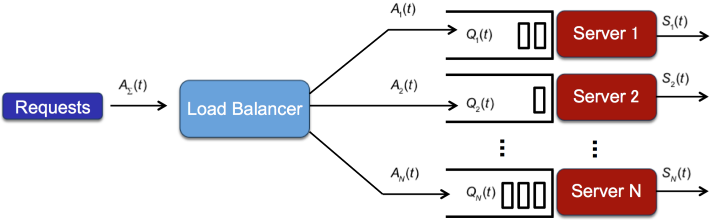
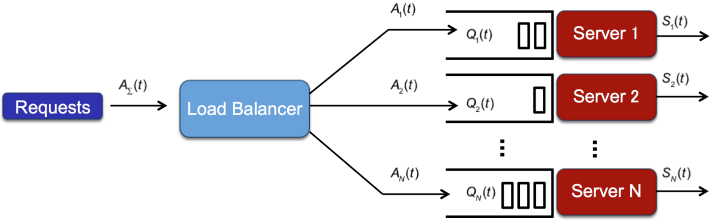


- All the N servers have the same average serve rate μ.

JBA in Homogeneous Servers



- All the N servers have the same average serve rate μ.
- The load balancer, under the JBA policy at each time-slot, randomly chose a queue among the queues that have workload less than the average workload at that time slot, and then forward all the incoming requests to that server.

S1 is Satisfied

- Let us first check (S1) by the choice $T = 1$:

$$\begin{aligned}
\mathbb{E}\left[\langle \mathbf{Q}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}\right] &= \langle \mathbf{Q}, \mathbb{E}\left[\mathbf{A}|\mathbf{Q}\right]\rangle - \langle \mathbf{Q}, \boldsymbol{\mu}\rangle \\
&= \frac{\lambda_\Sigma}{L}\sum_{n=1}^{L} Q_n - \mu \sum_{n=1}^{N} Q_n \\
&= (\frac{\lambda_\Sigma}{N} - \mu)\sum_{n=1}^{N} Q_n - (\frac{\lambda_\Sigma}{N} - \frac{\lambda_\Sigma}{L})\sum_{n=1}^{N} - \frac{\lambda}{N}\sum_{n=L+1}^{N} Q_n \\
&\leq -\frac{\epsilon}{N}\left\|\mathbf{Q}\right\|_1 - \lambda_\Sigma \frac{N-L}{N}(Q_{L+1} - Q_L) \\
&\leq -\frac{\epsilon}{N}\left\|\mathbf{Q}\right\|
\end{aligned} \tag{6}$$

assume $Q_1(t) \leq Q_2(t) \leq \cdots \leq Q_L(t) \leq Q^*(t) < Q_{L+1}(t) \leq \cdots \leq Q_N(t)$, and $Q^*(t) = \frac{1}{N}\sum Q_n(t)$ is the average queue length.

S1 is satisfied, and hence JBA is throughput optimal

S2 is Satisfied

- Let us turn to check (S2) with the line $\mathbf{c}$ as the projection direction:

$$\begin{aligned}
\mathbb{E}\left[\langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t) = \mathbf{Q}\right] &\overset{(a)}{=} \langle \mathbf{Q}_\perp, \mathbb{E}\left[\mathbf{A}|\mathbf{Q}\right]\rangle - \langle \mathbf{Q}_\perp, \boldsymbol{\mu}\rangle \\
&= \langle \mathbf{Q}_\perp, \mathbb{E}\left[\mathbf{A}|\mathbf{Q}\right]\rangle \\
&= \frac{\lambda_\Sigma}{L}\sum_{n=1}^{L}(Q_n - Q^*) \\
&= -\frac{\lambda_\Sigma}{L}\sum_{n=1}^{L}|Q_n - Q^*| \\
&\leq -\frac{\lambda_\Sigma}{2N}\|\mathbf{Q}_\perp\| \\
&\leq -\frac{\mu}{2}\|\mathbf{Q}_\perp\|
\end{aligned} \tag{7}$$

for all $0 < \epsilon \leq \frac{N\mu}{2}$. (a) follows $\langle \mathbf{Q}_\perp, \mathbf{1}\rangle = 0$

S2 is verified and hence JBA is heavy-traffic optimality

Theorems

Theorem

For any λ_Σ *in the interior of* $\mathcal{R}$, *i.e.,* $\lambda_\Sigma < \mu_\Sigma$, *the JBA routing policy stabilizes the system, and all the moments of the stationary distribution are bounded, i.e., there exist finite constants* $\{M_r, r \in \mathbb{N}\}$ *such that* $\mathbb{E}\left[\left\|\overline{\mathbf{Q}}\right\|^r\right] \leq M_r$.

Theorem

Consider a set of load balancing system under JBA policy with the exogenous arrival process $\{A_\Sigma^{(\epsilon)}(t), t \geq 0\}$, *parameterized by* $\epsilon > 0$. *Then, each of these systems, the expectation of the sum queue length in steady state is lower bounded by*

$$\mathbb{E}\left[\sum_{n=1}^{N} \overline{Q}_n^{(\epsilon)}\right] \geq \frac{\zeta^{(\epsilon)}}{2\epsilon} - K \tag{8}$$

where $\zeta^{(\epsilon)} = (\sigma_\Sigma^{(\epsilon)})^2 + \nu_\Sigma^2 + \epsilon^2$, $K = \frac{NS_{\max}}{2}$. *Therefore, in the heavy-traffic limit as* $\epsilon \downarrow 0$, *assuming the* $(\sigma_\Sigma^{(\epsilon)})^2$ *converges to a constant* σ_Σ^2, *we have*

$$\liminf_{\epsilon \downarrow 0} \epsilon \mathbb{E}\left[\sum_{n=1}^{N} \overline{Q}_n^{(\epsilon)}\right] \geq \frac{\zeta}{2}, \tag{9}$$

Theorems (Cont'd)

Theorem

Consider a set of load balancing system with the exogenous arrival process $\{A_\Sigma^{(\epsilon)}(t), t \geq 0\}$*, parameterized by* $\epsilon > 0$*, such that the mean arrival rate is* $\lambda_\Sigma^{(\epsilon)} = \mu_\Sigma - \epsilon$ *and variance is denoted as* $(\sigma_\Sigma^{(\epsilon)})^2$*. Under the JBA algorithm,* $\{\mathbf{Q}^{(\epsilon)}(t), t \geq 0\}$ *converges in distribution to* $\overline{\mathbf{Q}}^{(\epsilon)}$*. Then, there exist finite constants* $\{M_r, r \in \mathbb{N}\}$ *which are independent of* ϵ *such that for all* $r \in \mathbb{N}$*,*

$$\mathbb{E}\left[\left\|\overline{\mathbf{Q}}_\perp^{(\epsilon)}\right\|\right] \leq M_r, \tag{10}$$

for all system with $0 < \epsilon \leq \frac{N\mu}{2}$*.*

Theorems (Cont'd)

Theorem

Consider a set of load balancing system with the exogenous arrival process $\{A_{\Sigma}^{(\epsilon)}(t), t \geq 0\}$, *parameterized by* $\epsilon > 0$, *such that the mean arrival rate is* $\lambda_{\Sigma}^{(\epsilon)} = \mu_{\Sigma} - \epsilon$ *and variance is denoted as* $(\sigma_{\Sigma}^{(\epsilon)})^2$. *Under the JBA algorithm,* $\{\mathbf{Q}^{(\epsilon)}(t), t \geq 0\}$ *converges in distribution to* $\overline{\mathbf{Q}}^{(\epsilon)}$. *For each system with* $0 < \epsilon \leq \frac{N\mu}{2}$, *the steady state average queue length satisfies*

$$\mathbb{E}\left[\sum_{n=1}^{N} \overline{Q}_n^{(\epsilon)}\right] \leq \frac{\zeta^{(\epsilon)}}{2\epsilon} + \overline{B}^{(\epsilon)}, \tag{11}$$

where $\zeta^{(\epsilon)}$ *is the same as in the lower bound, and* $\overline{B}^{(\epsilon)}$ *is* $o(\frac{1}{\epsilon})$, *i.e.,* $\lim_{\epsilon \downarrow 0} \epsilon \overline{B}^{(\epsilon)} = 0$.

Therefore, assuming the variance $(\sigma_{\Sigma}^{(\epsilon)})^2$ *converges to a constant* σ_{Σ}^2, *the upper bound becomes*

$$\limsup_{\epsilon \downarrow 0} \epsilon \mathbb{E}\left[\sum_{n=1}^{N} \overline{Q}_n^{(\epsilon)}\right] \leq \frac{\zeta}{2} \tag{12}$$

How about Random Load Balancing

Under random load balancing, we have $\mathbf{A} - \mathbf{S} = -\frac{\epsilon}{N}\mathbf{1}$

- For (S1), we have $\mathbb{E}\left[\langle \mathbf{Q}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}\right] = -\frac{\epsilon}{N}\left\|\mathbf{Q}\right\| \leq -\frac{\epsilon}{2N}\left\|\mathbf{Q}\right\|$
- For (S2), we have $\mathbb{E}\left[\langle \mathbf{Q}_{\perp}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t) = \mathbf{Q}\right] = 0$ for all t

Consider steady-state collapse, random load balancing actually does no harm in the sense that it would not incur any positive drift.

Can we utilize this fact to turn bad to good?

From 1 to any finite T

The load balancer uses JBA, Power of d, or JSQ every T time-slots, otherwise just random routing.

- For (S1), it is trivial to hold.
- For (S2), by letting $T_2 = T$, we have

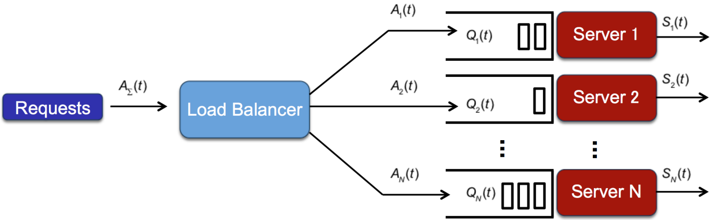
$$
\begin{aligned}
&\mathbb{E}\left[\sum_{t=t_0}^{t_0+T_2-1} \langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&= \sum_{t=t_0}^{t_0+T_2-1} \mathbb{E}\left[\langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&\overset{(a)}{=} \sum_{t=t_0}^{t_0+T_2-1} \mathbb{E}\left[\mathbb{E}\left[\langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t)\right] | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&= \mathbb{E}\left[-\eta \left\|\mathbf{Q}_\perp(t^*)\right\| | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&\leq -\eta \left\|\mathbf{Q}_\perp(t_0)\right\| + DT
\end{aligned}
\tag{13}
$$

where (a) follows from tower property of conditional expectation.

Outline

1. Introduction and Motivation
   - Why do we need an effective and fast cloud?
   - Load Balancing and Previous Works
2. Throughput and Heavy-traffic optimality of General Load Balancing
   - Model, Challenges, Contributions and Key insights
   - Methodology and Sufficient Conditions
   - Homogeneous Servers
   - Heterogeneous Servers
   - Conclusions

JBA in Heterogeneous Servers



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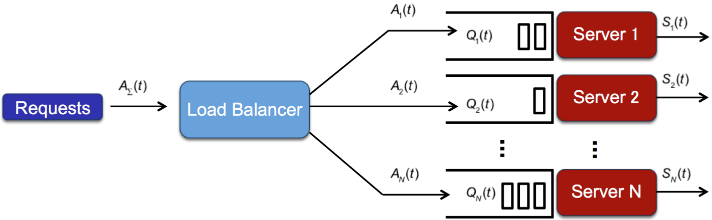

- The N servers do not have the same average serve rate, each with μ_n.

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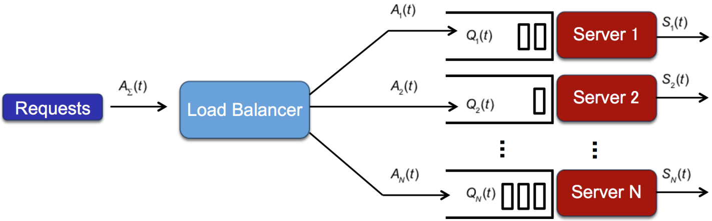

- The N servers do not have the same average serve rate, each with μ_n.
- The load balancer, under the JBA policy at each time-slot, randomly with probability proportional to the service rate μ_n to chose a queue among the queues that have workload less than the average workload at that time slot, and then forward all the incoming requests to that server.

$$\mathbb{P}(R_i) = \frac{\mu_i}{\sum_{i \leq L} \mu_i} \quad \text{if} \quad i \leq L \tag{14}$$

JBA in Heterogeneous Servers (Cont'd)

Under the JBA load balancing, it can be shown that (S1) and (S2) still hold, i.e.,

- For (S1), we have $\mathbb{E}\left[\langle \mathbf{Q}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}\right] \leq -\delta \left\| \mathbf{Q} \right\|$ for some $\delta > 0$
- For (S2), we have $\mathbb{E}\left[\langle \mathbf{Q}_{\perp}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t) = \mathbf{Q}\right] \leq -\eta \left\| \mathbf{Q}_{\perp}(t) \right\|$ for some $\eta > 0$ independent of ϵ

Under purely random load balancing with proportional probability, we have

- For (S1), we have $\mathbb{E}\left[\langle \mathbf{Q}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}\right] \leq -\delta \left\| \mathbf{Q} \right\|$ for some $\delta > 0$
- For (S2), we have $\mathbb{E}\left[\langle \mathbf{Q}_{\perp}(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t) = \mathbf{Q}\right] \leq \epsilon \left\| \mathbf{Q}_{\perp}(t) \right\|$ for all t

From 1 to any finite T

The load balancer uses JBA, Power of d, or JSQ every T time-slots, otherwise just random routing with proportional probability over N servers.

- For (S1), it is trivial to hold.
- For (S2), by letting $T_2 = T$, we have

$$\begin{aligned}
&\mathbb{E}\left[\sum_{t=t_0}^{t_0+T_2-1} \langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&= \sum_{t=t_0}^{t_0+T_2-1} \mathbb{E}\left[\langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&= \sum_{t=t_0}^{t_0+T_2-1} \mathbb{E}\left[\mathbb{E}\left[\langle \mathbf{Q}_\perp(t), \mathbf{A}(t) - \mathbf{S}(t)\rangle | \mathbf{Q}(t)\right] | \mathbf{Q}(t_0) = \mathbf{Q}\right] \\
&\le ((T-1)\epsilon - \eta)\left\|\mathbf{Q}_\perp(t_0)\right\| + DT^2 \\
&\le -\frac{1}{2\eta}\left\|\mathbf{Q}_\perp(t_0)\right\| + DT^2
\end{aligned} \tag{15}$$

for all $0 < \epsilon \le \frac{\eta}{2T}$, hence smaller ϵ means a larger T!.

Outline

Conclusions

- Can we generalize existing load balancing algorithm?
  - Yes, the proposed JBA policy
- Do we really need sampling for each time-slot for heavy-traffic optimality?
  - No, we can actually sampling every T slots whenever $T\epsilon \leq \alpha$ is satisfied.

Thank you!

Q & A