On Differentially Private Federated Linear Contextual Bandits

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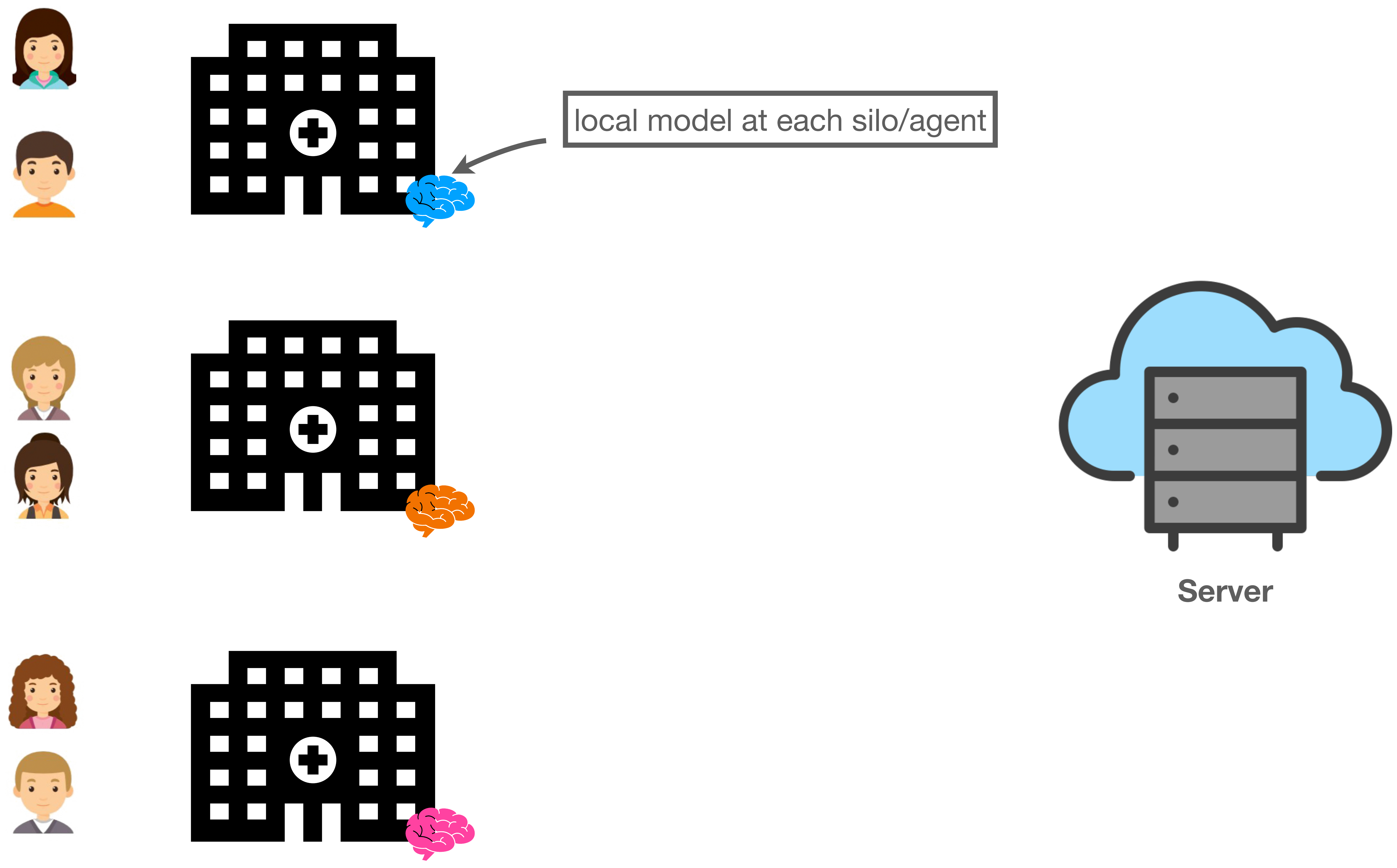
Hosted by Prof. Guoliang Xue

Joint work with **Sayak Ray Chowdhury**, Microsoft Research, India

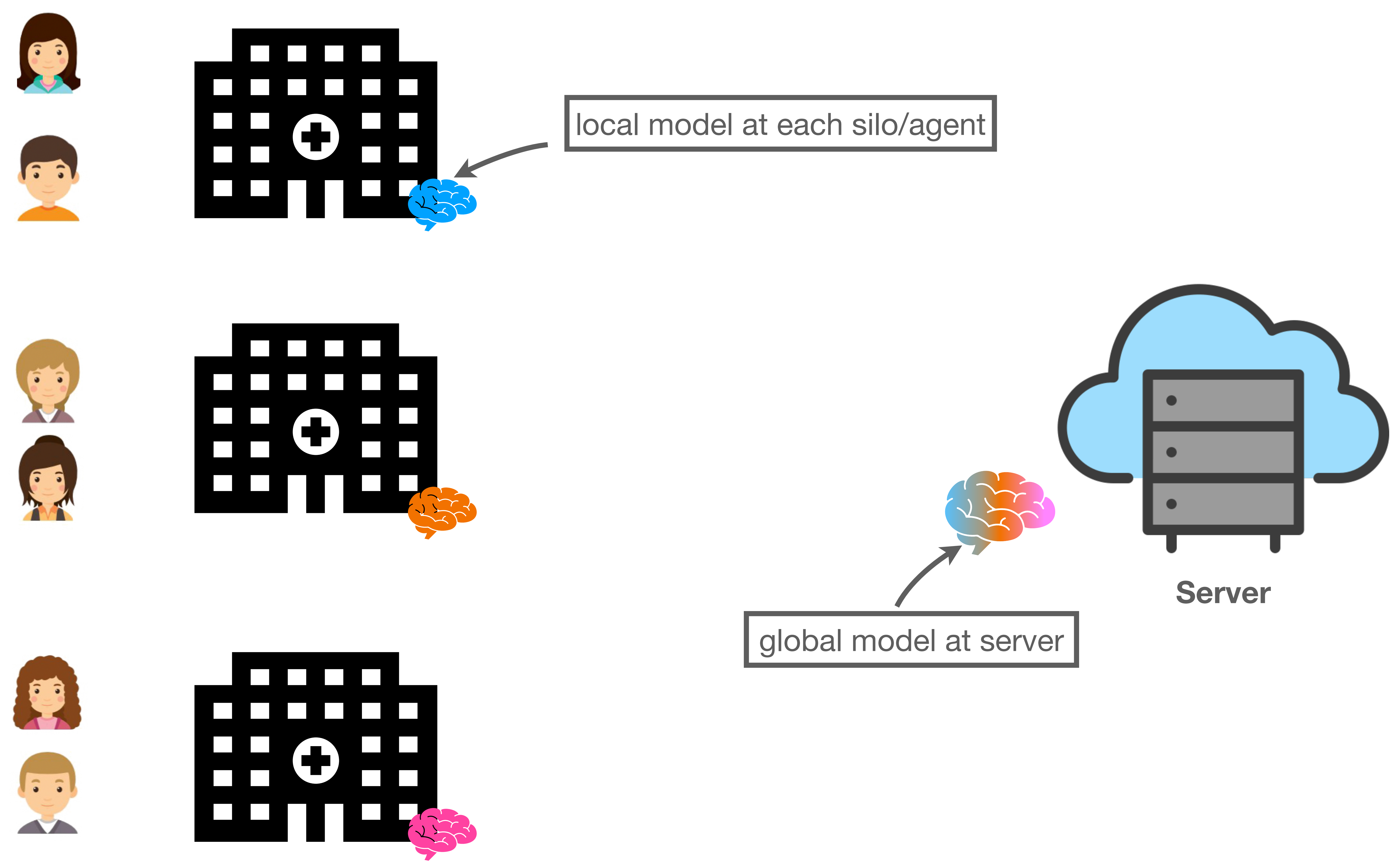
Cross-silo Federated Learning[KMA+19]

A hospital example



Cross-silo Federated Learning

A hospital example



Cross-silo Federated Learning

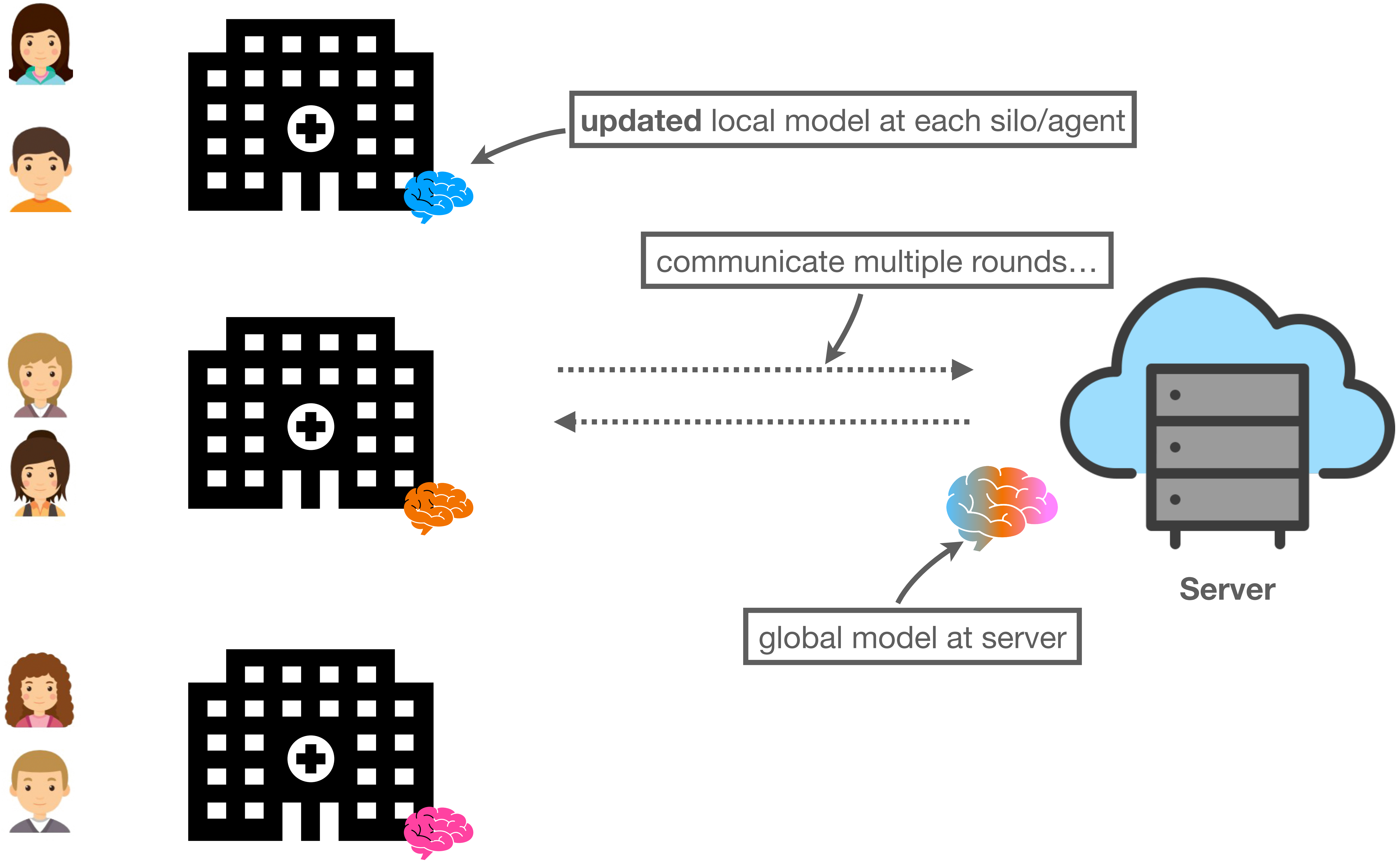
A hospital example

Cross-device FL

- Large no. of clients
- Limited resource
- e.g., clients are phones

Cross-silo FL

- Small no. agents/silos
- More resource
- e.g., silos are hospitals, banks, schools

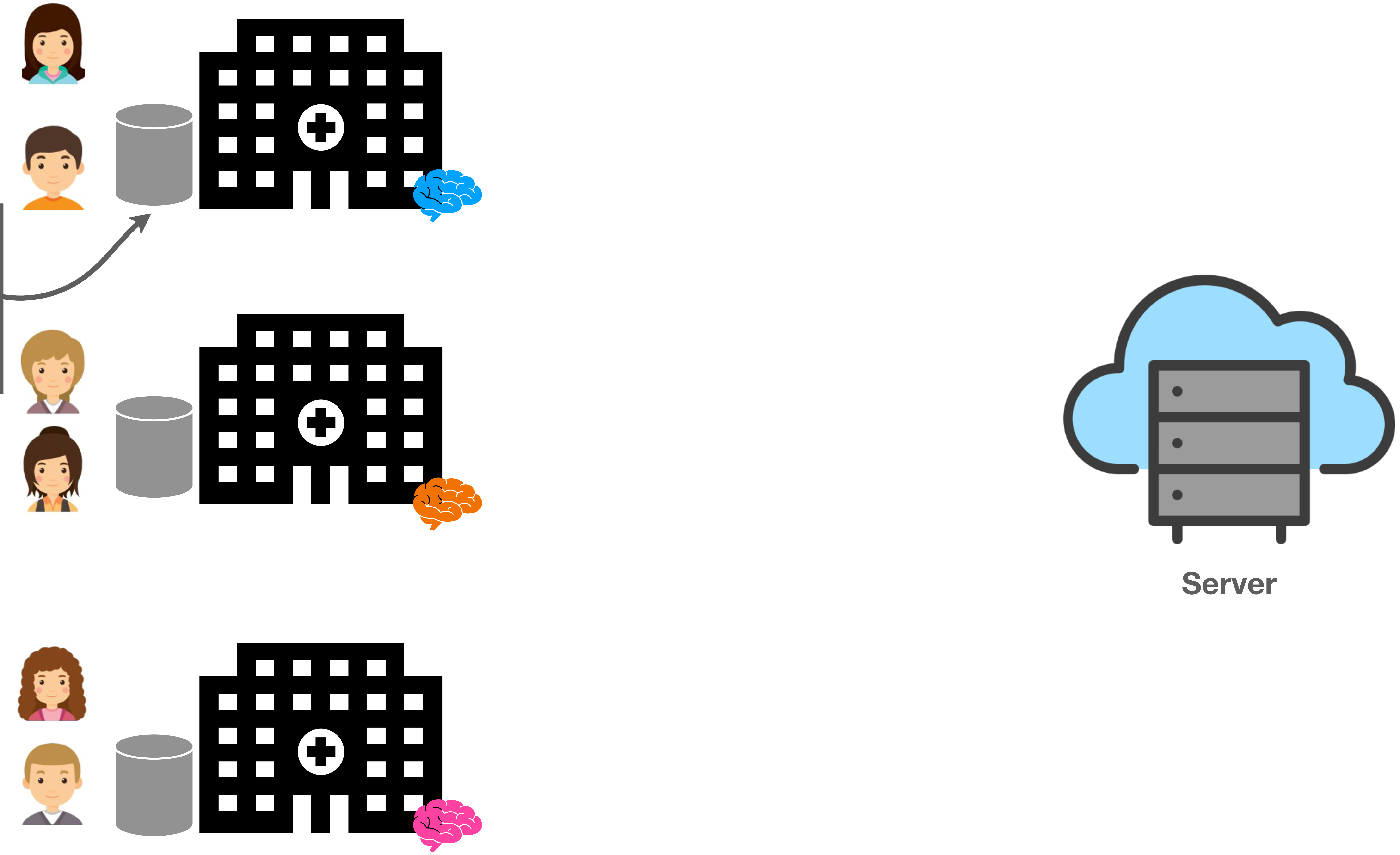


Cross-silo Federated Learning

Batch vs. Online learning

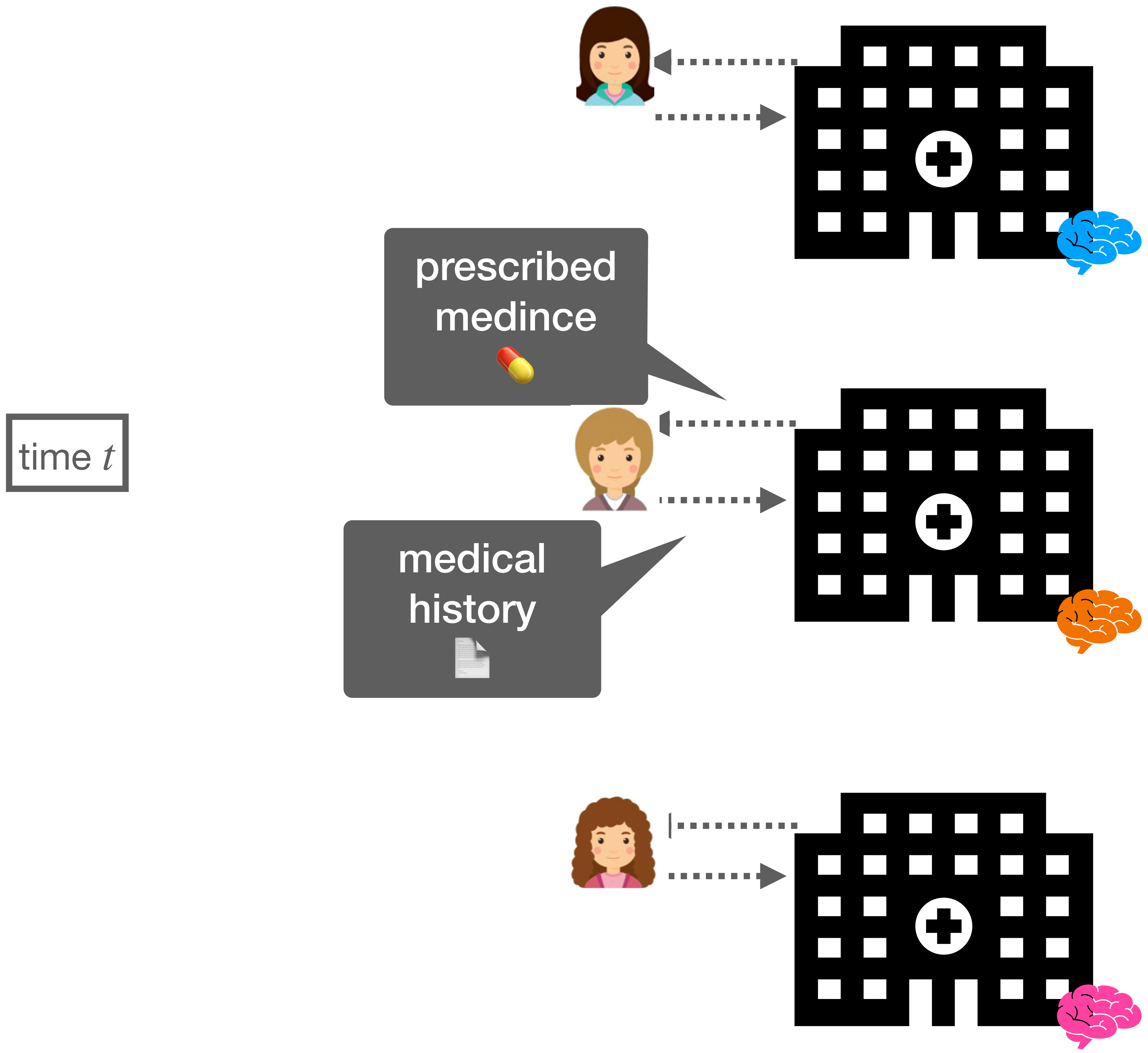
In FL **batch (supervised)** learning:

- each silo has a static dataset
- offline training
- e.g., cancer diagnosis



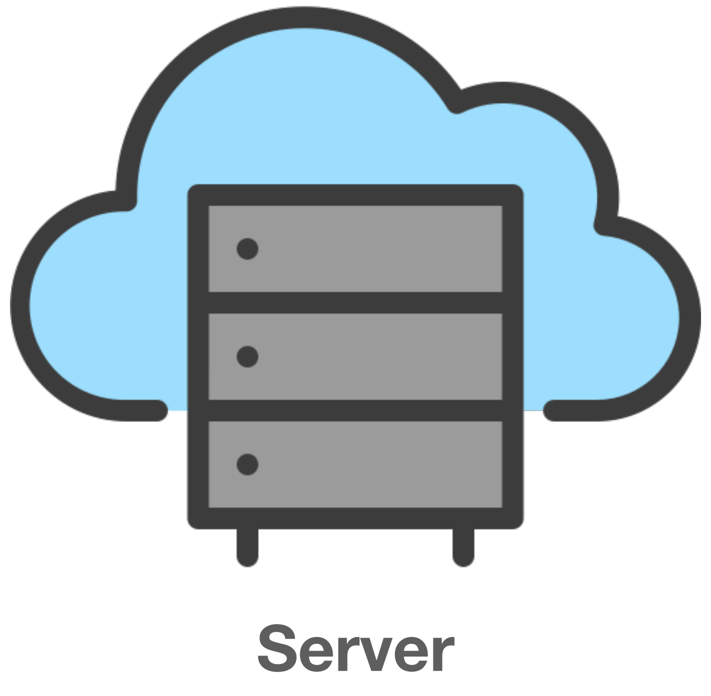
Cross-silo Federated Learning

Batch vs. Online learning



In FL **online** learning:

- each silo has a stream of data
- online training/decision, i.e., learn from interaction
- e.g., personalized medical care



Cross-silo Federated Learning

Batch vs. Online learning

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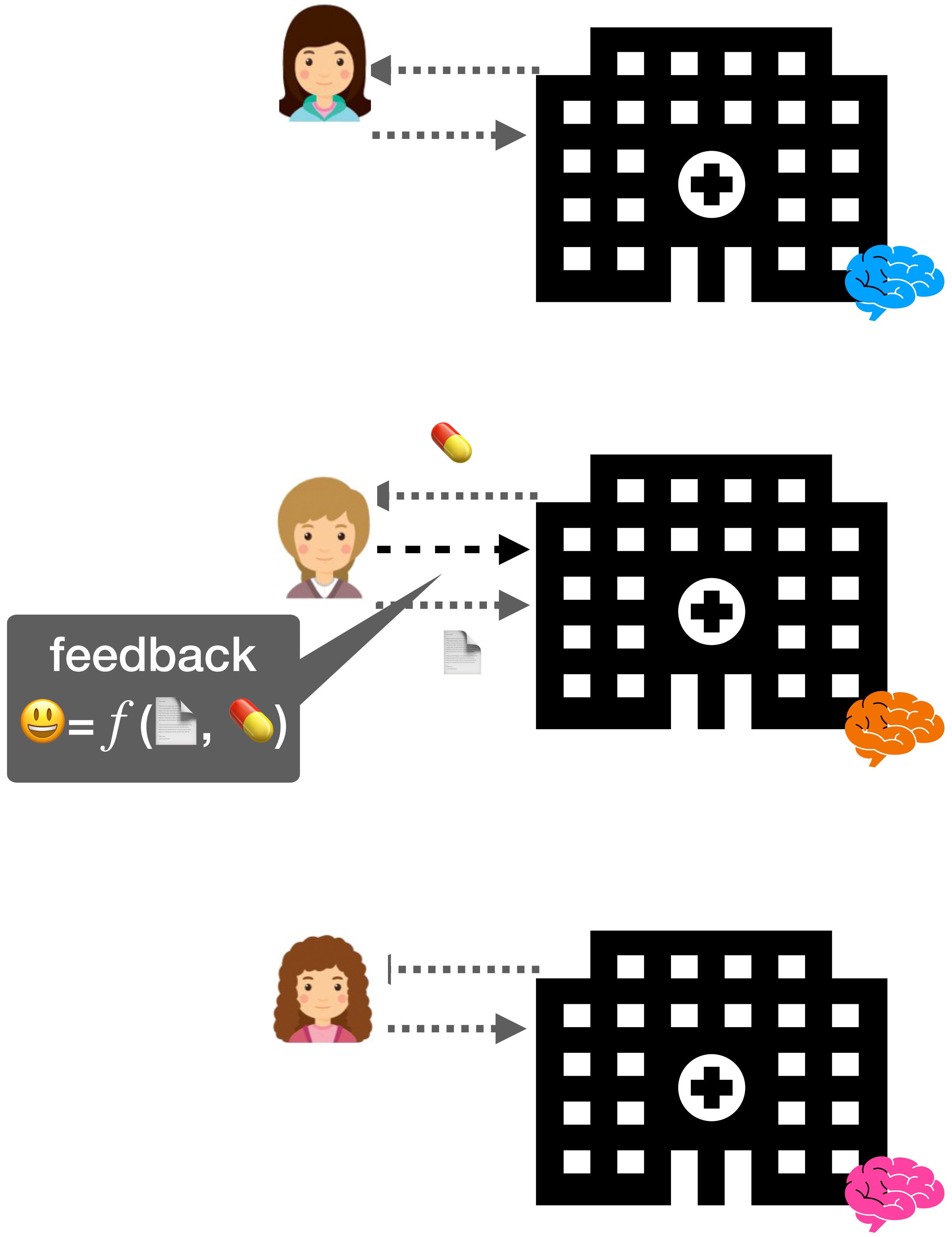
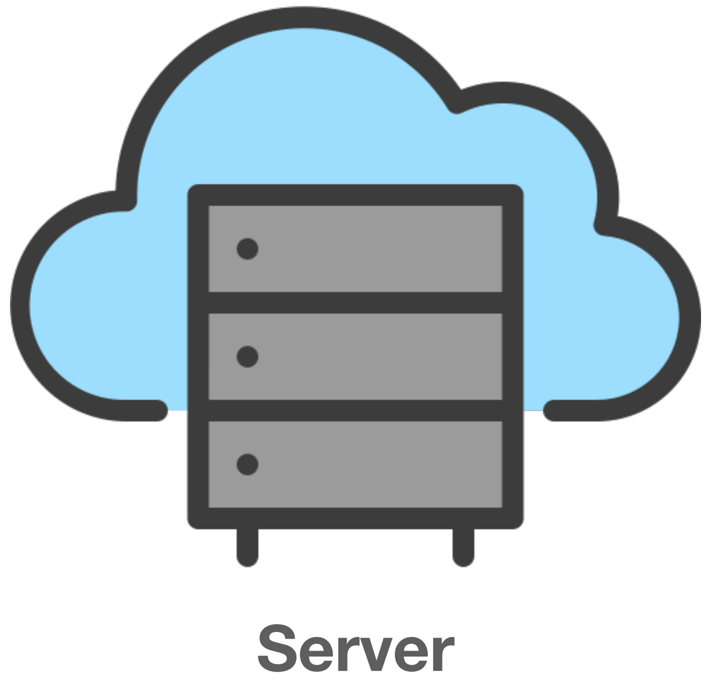
time t

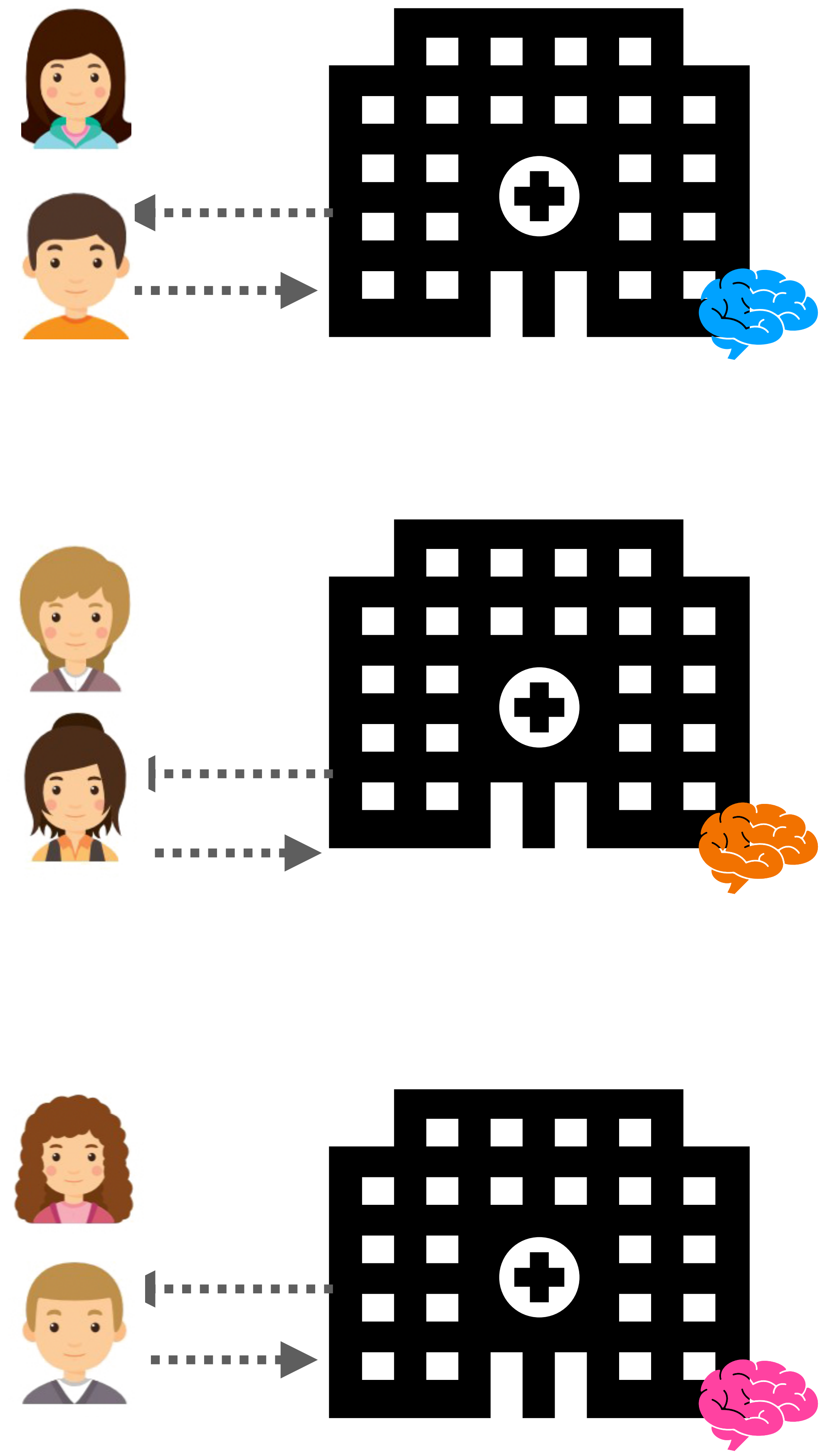




Cross-silo Federated Learning

Batch vs. Online learning

time $t + 1$



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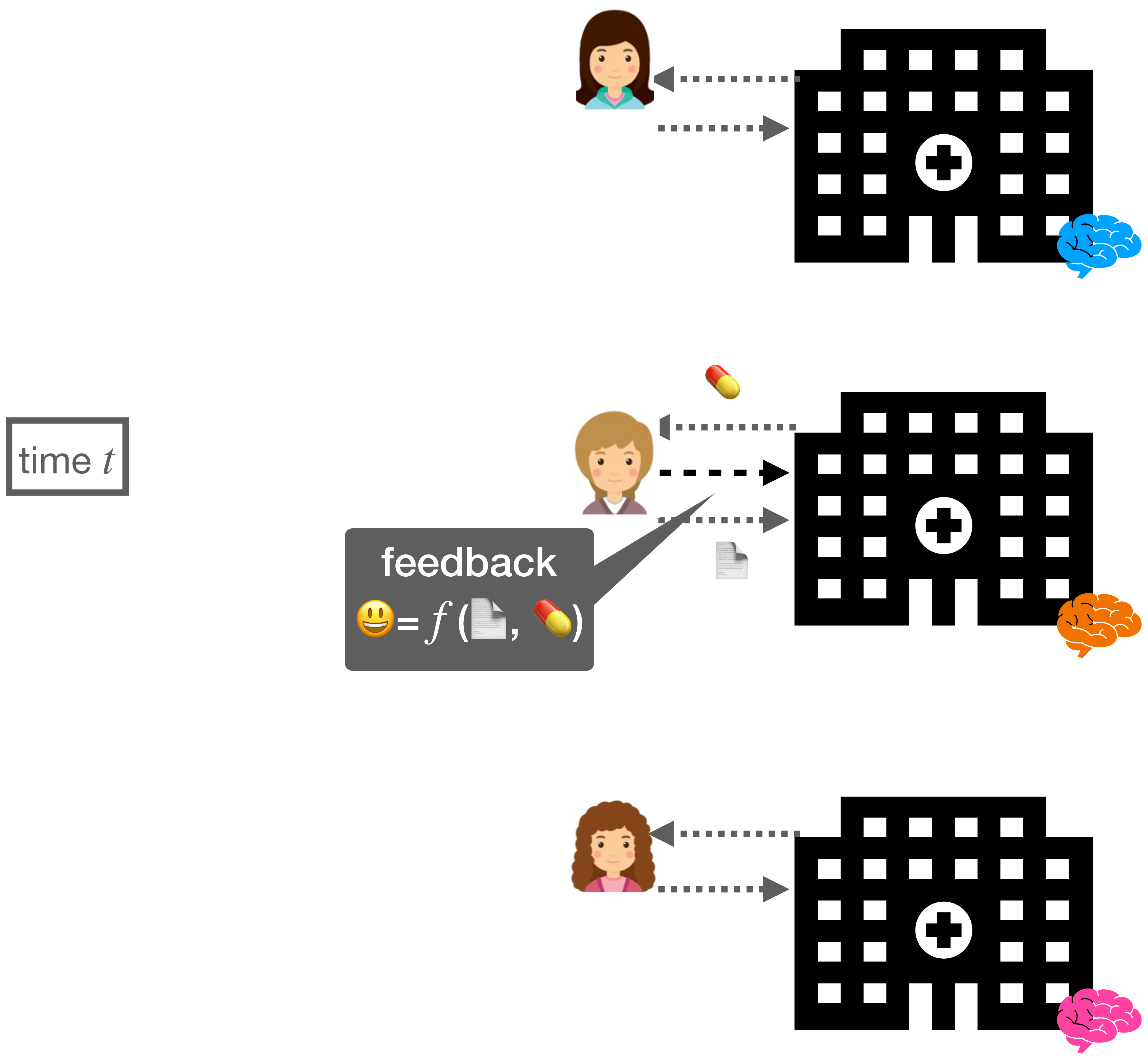
Cross-silo Federated Learning

Linear contextual bandits (LCB)[APS11]



In FL **LCBs** (online):

- unknown reward feedback f is a **linear** function $y_t = x_t^\top \theta^* + \eta_t$
- $x_t = \phi(c_t, a_t) \in \mathbb{R}^d$, ϕ is the feature map, c_t is context and a_t is the action
- θ^* is the unknown parameter
- η_t is zero-mean noise

Server

Performance metric: **group regret** over M agents during T rounds

$$R_M(T) = \sum_{i=1}^{M} \sum_{t=1}^{T} \left[\max_a \phi_i(c_{t,i}, a)^\top \theta^* - \phi_i(c_{t,i}, a_{t,i})^\top \theta^* \right]$$

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On Differentially Private Federated Linear Contextual Bandits

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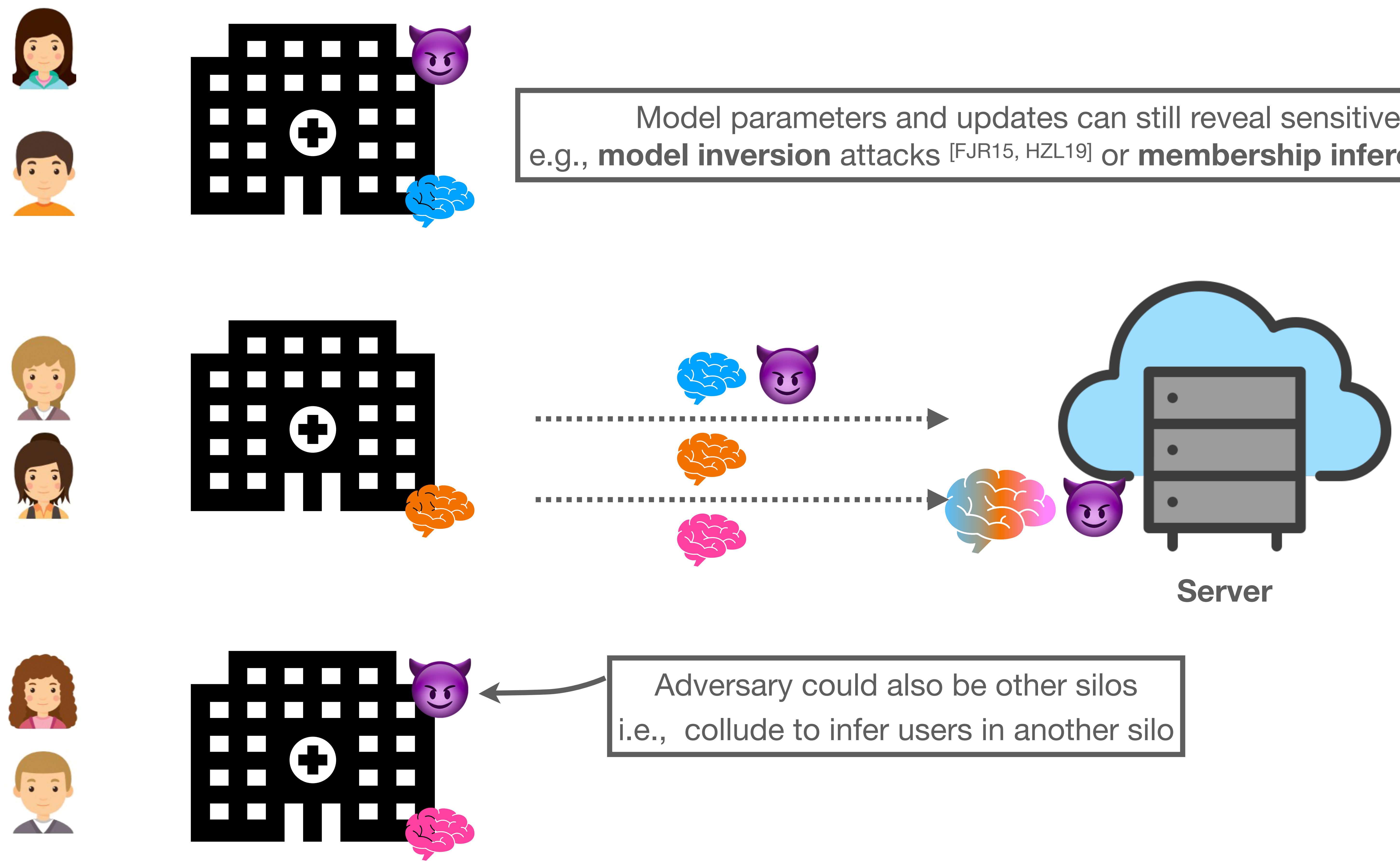
Privacy in Cross-silo FL

Though locally stored data, privacy risks still exist



Model parameters and updates can still reveal sensitive information
e.g., **model inversion** attacks [FJR15, HZL19] or **membership inference** attacks [SSS+17]

Adversary could also be other silos
i.e., collude to infer users in another silo

Differentially Private Cross-silo FL

Differential privacy[DR14] — a rigorous privacy protection

Differential Privacy 101

Definition. If for any two neighboring datasets D and D', and any outcome E

$$\mathbb{P}(M(D) \in E) \leq e^{\epsilon}\mathbb{P}(M(D') \in E) + \delta$$

Then, M satisfies (ϵ, δ)-DP

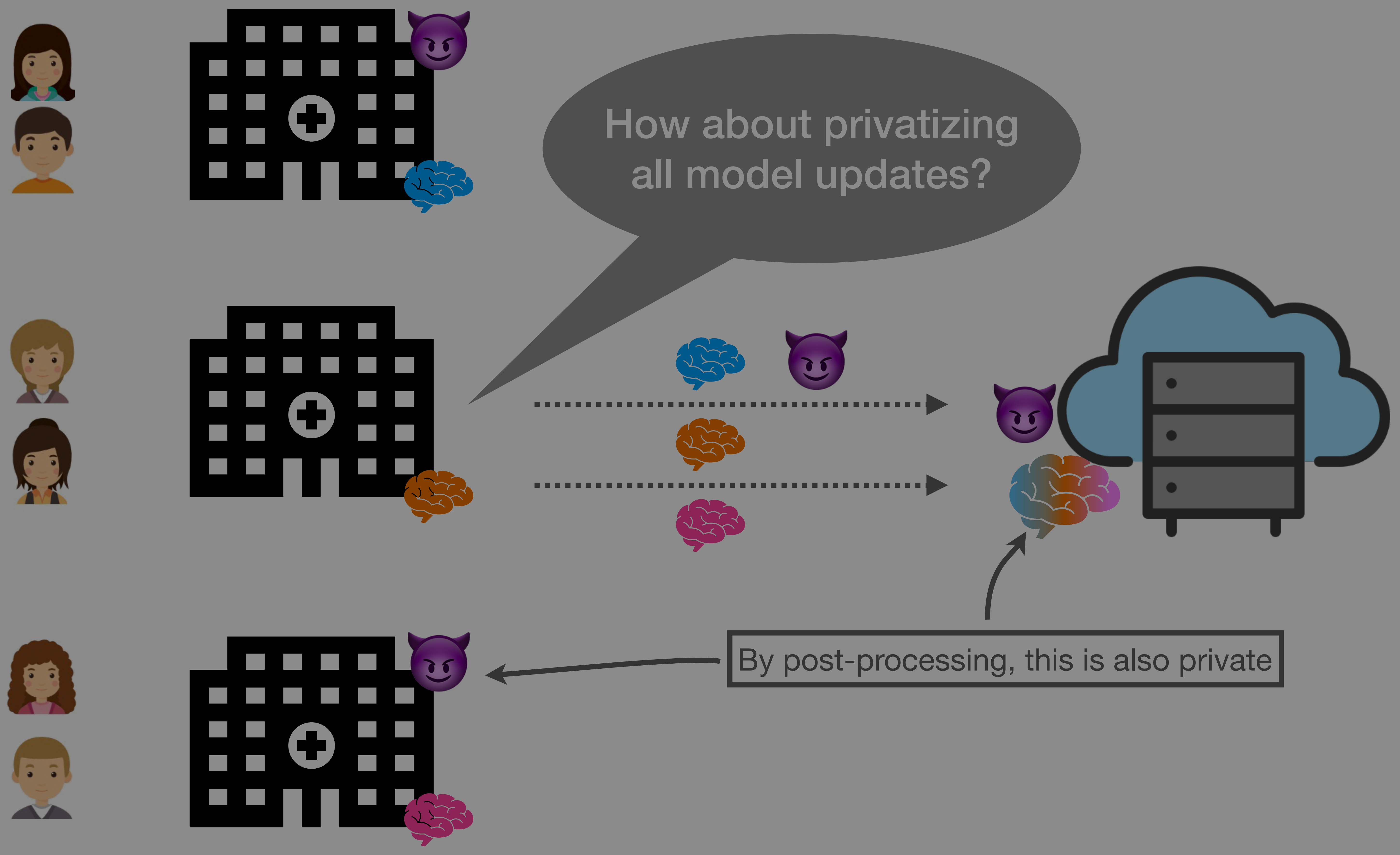
— DP means that outputs are "close" in probability[1] on two neighboring datasets

Key components:

1. *What are the neighboring datasets?*
   — the identity for protection
2. *What are the outputs?*
   — the view of adversary

Key properties:

1. *Composition*, privacy loss adds up
2. *Post-processing,* immune to further processing if data is not touched



[1] A more general view is via f-divergence or Renyi divergence between two distributions

Silo-level Local Differential Privacy (LDP)

All communication from each silo is private

Silo-level LDP [1]

Definition (informal). The full transcript of communication between any agent $i \in [M]$ and server are "close" in prob. on any two local neighboring datasets at agent i

Local neighboring datasets at agent i*: a sequence of* T *users that differs in only one user*

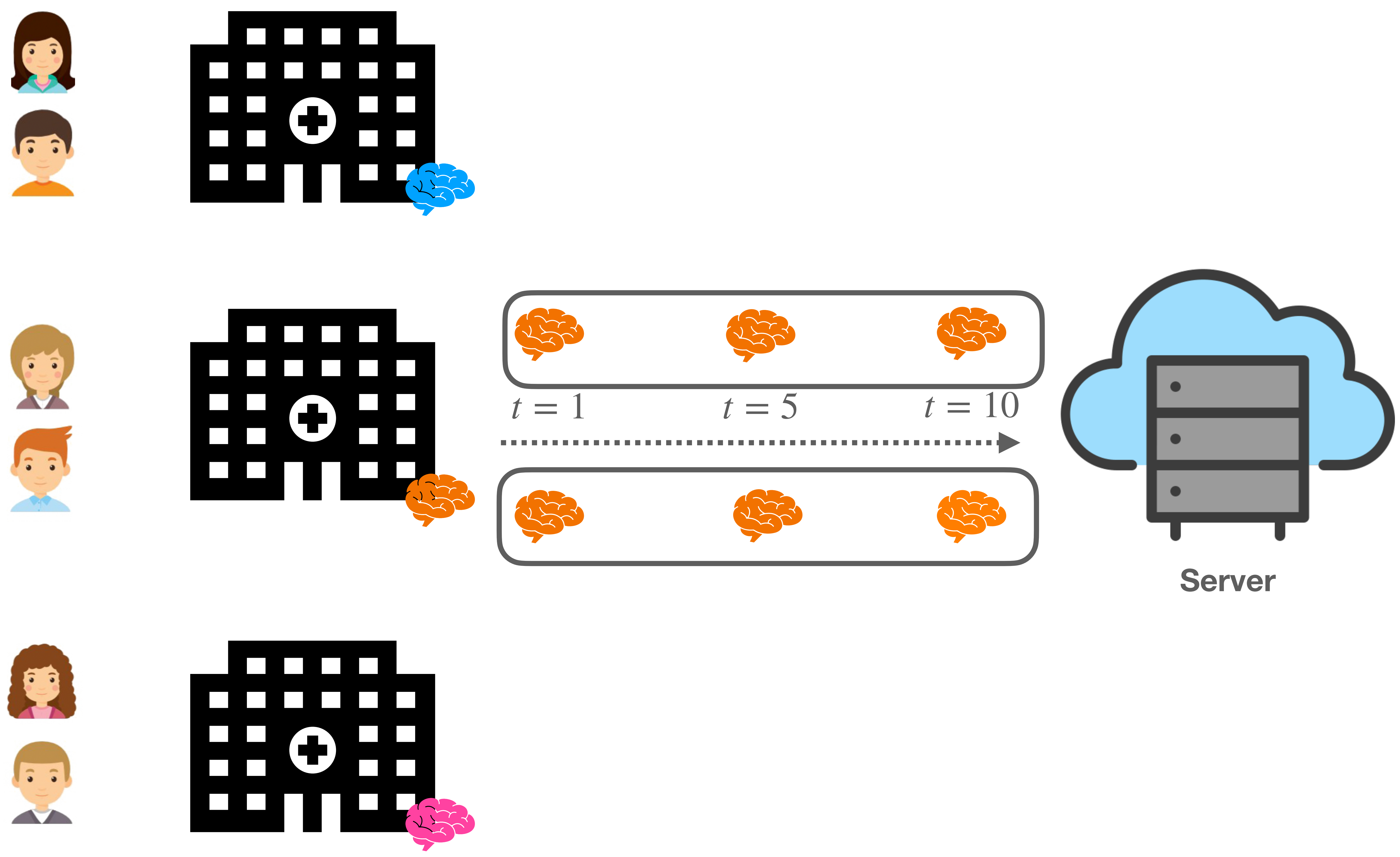
— protect each user/patient

— different from standard DP for cross-device FL, where each client is protected

Outputs: *full communication transcript*

— communicated models/messages

— communication schedule, i.e., when communication happens



[1] Similar privacy notion in different names exist in previous works, see [LR21, LHW+22]

On Differentially Private Federated Linear Contextual Bandits

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State-of-the-Art

Private Federated LCB

The state-of-the-art[DP20]

[Dubey&Pentland '20]

Algorithm: federated LinUCB with Gaussian mechanism (tree-based)

Privacy: essentially the same as silo-level LDP

Regret: *additional* regret due to privacy is $\tilde{O}(\sqrt{MT/\epsilon})$

Communication rounds: $O(\log T)$

Conclusion: match the regret achieved by a "super" single agent

Fundamental Issues

Privacy leakage

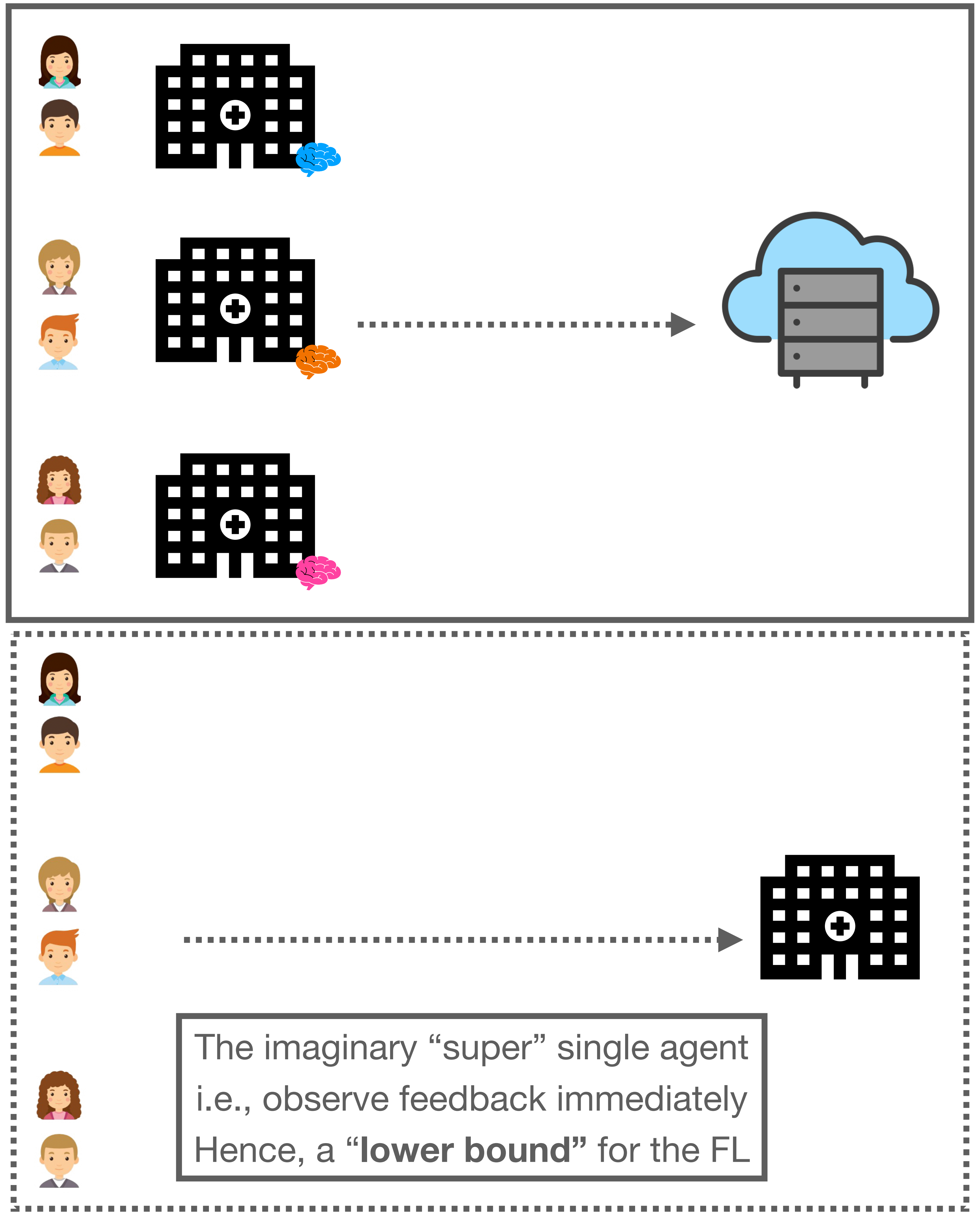
- The proposed algorithm fails to guarantee silo-level LDP
- A simple attack can reveal sensitive information of users

Incorrect regret

- The claimed privacy cost is mis-calculated
- The correct one is $\tilde{O}(M^{3/4}\sqrt{T/\epsilon})$
- Hence, no longer match the "lower bound"

Ungrounded communication cost

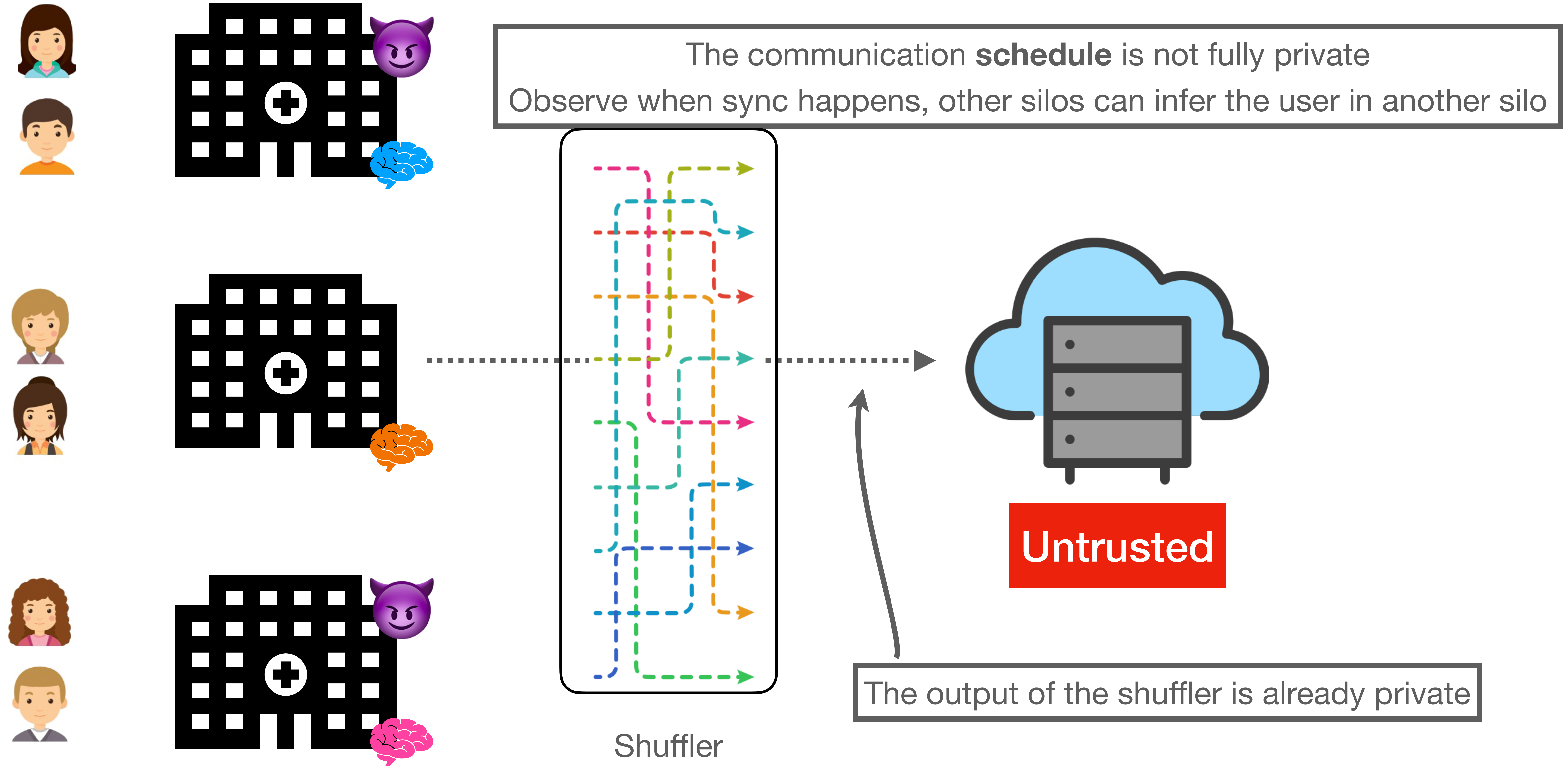
- The claimed $O(\log T)$ rounds comm. cost is not grounded



[DP20] Differentially-private federated linear bandits. *NeurIPS* 2020

Contribution

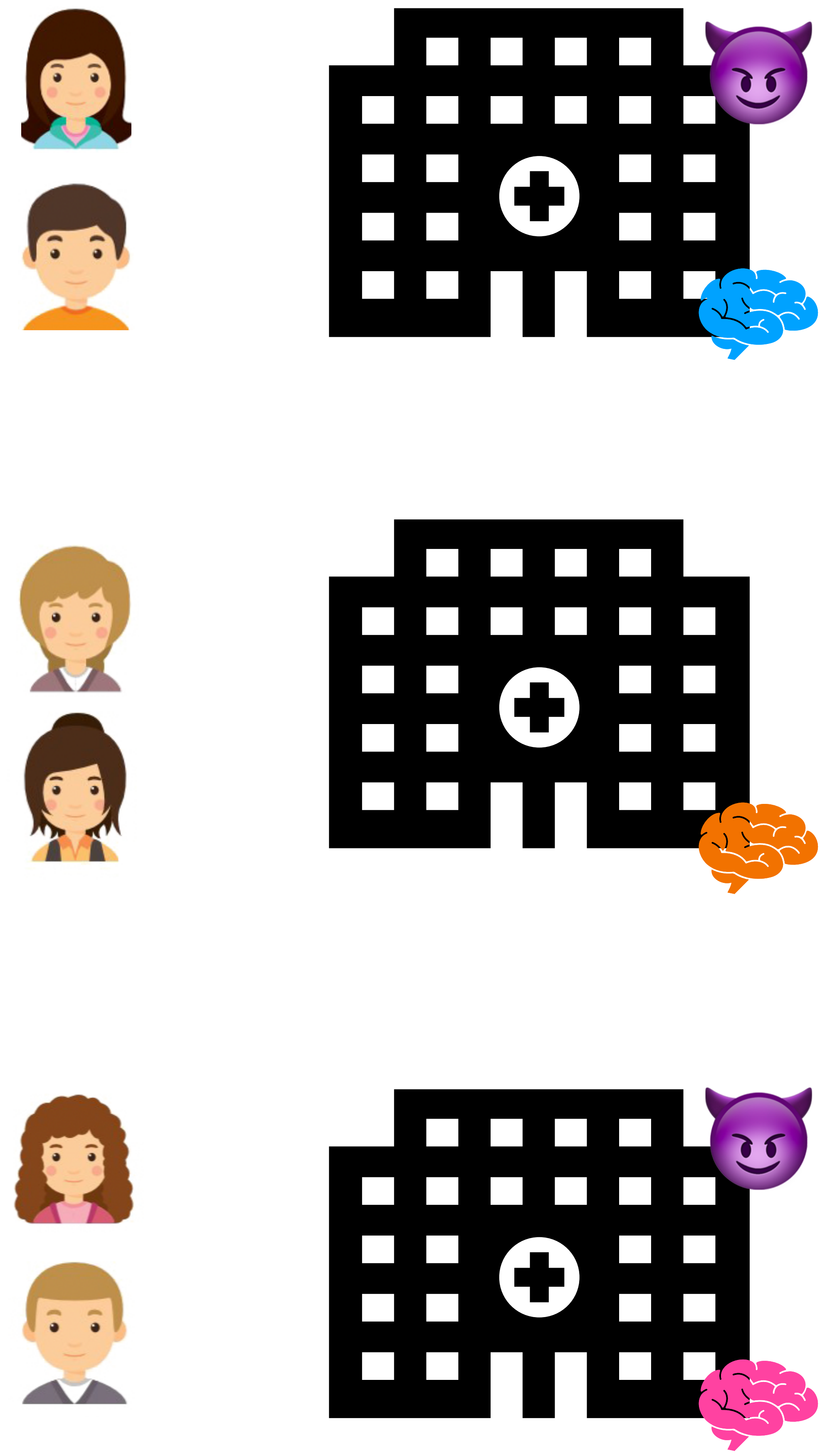
Main Results



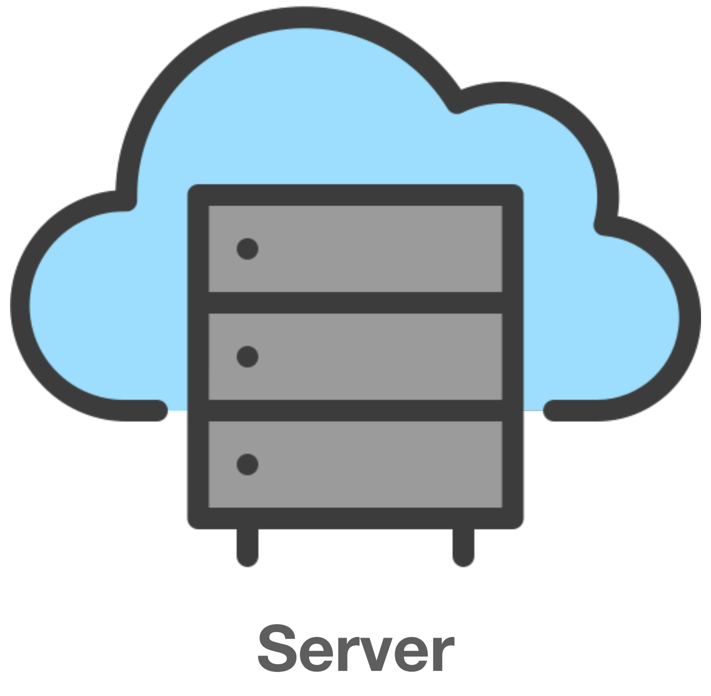
1. Identify the privacy, regret and communication cost gaps in the state-of-the-art
2. Propose a generic federated algorithm with flexible privacy protocols
3. Achieve the first correct regret bound under **silo-level LDP**, i.e., the privacy cost is $\tilde{O}(M^{3/4}\sqrt{T/\epsilon})$
4. Shave the additional $M^{1/4}$ factor under **shuffle differential privacy (SDP)** — still a strong privacy model

Privacy Gap in SOTA

Dynamic communication leaks privacy



The communication **schedule** is not fully private
Observe when sync happens, other silos can infer the user in another silo



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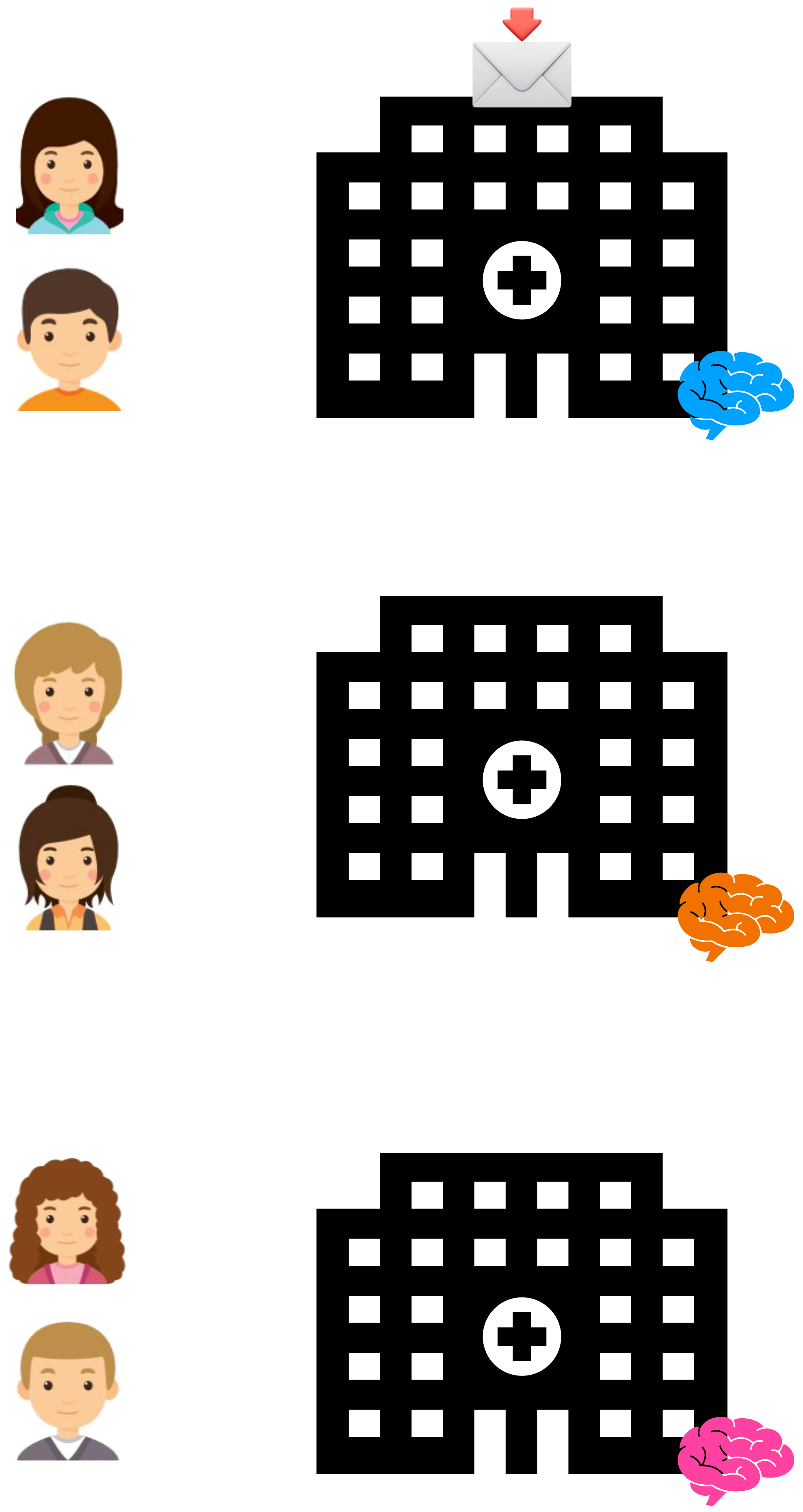


Communication schedule for silos in SOTA

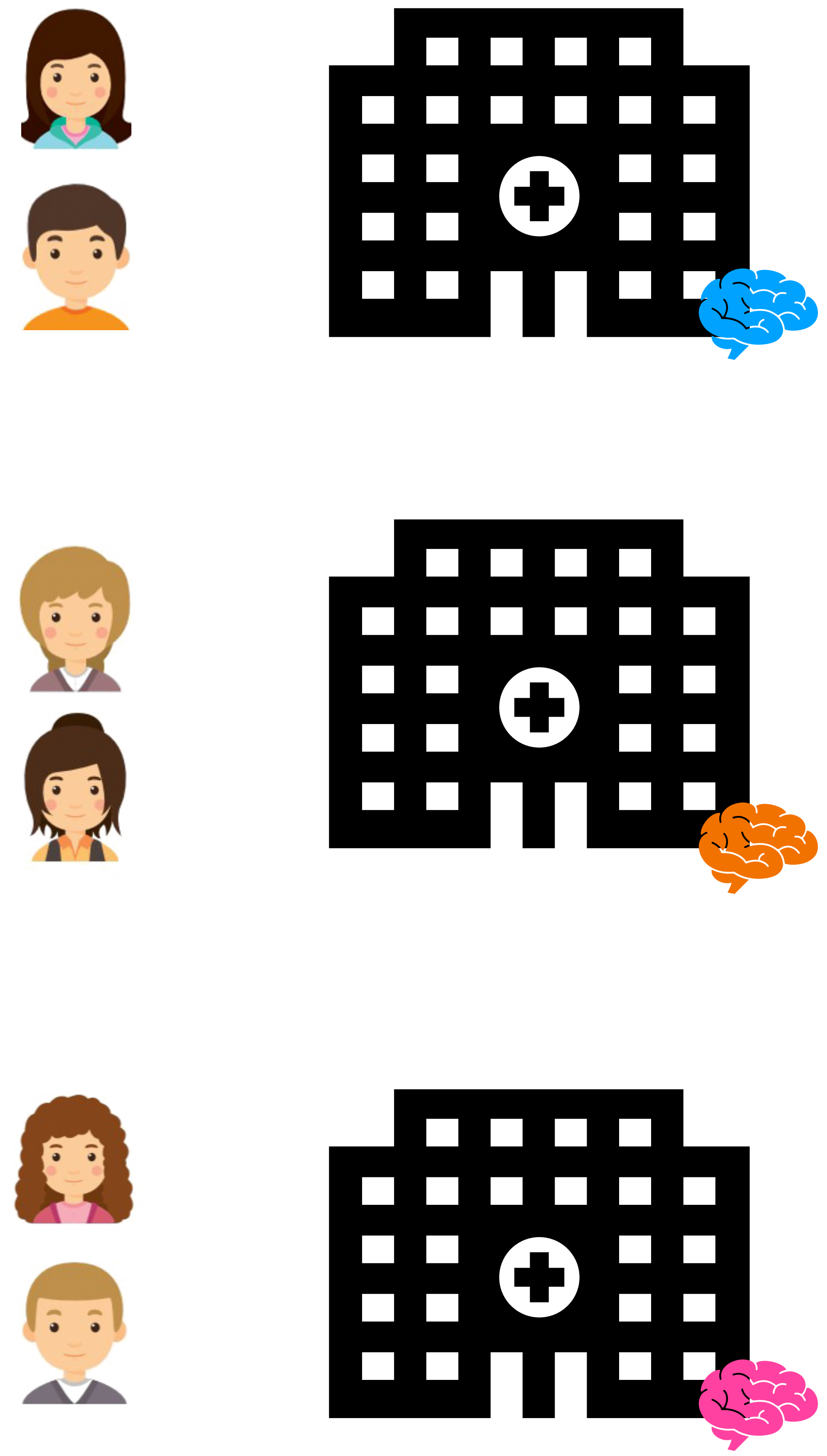
$$\exists i \in [M], \quad f(X_i, Z) > 0$$

- Z — all previous sync data among all silos
- X_i — **new non-private** local data at silo i since recent sync
- f — sync function, shared among all silos

Server

Privacy Gap in SOTA

Dynamic communication leaks privacy



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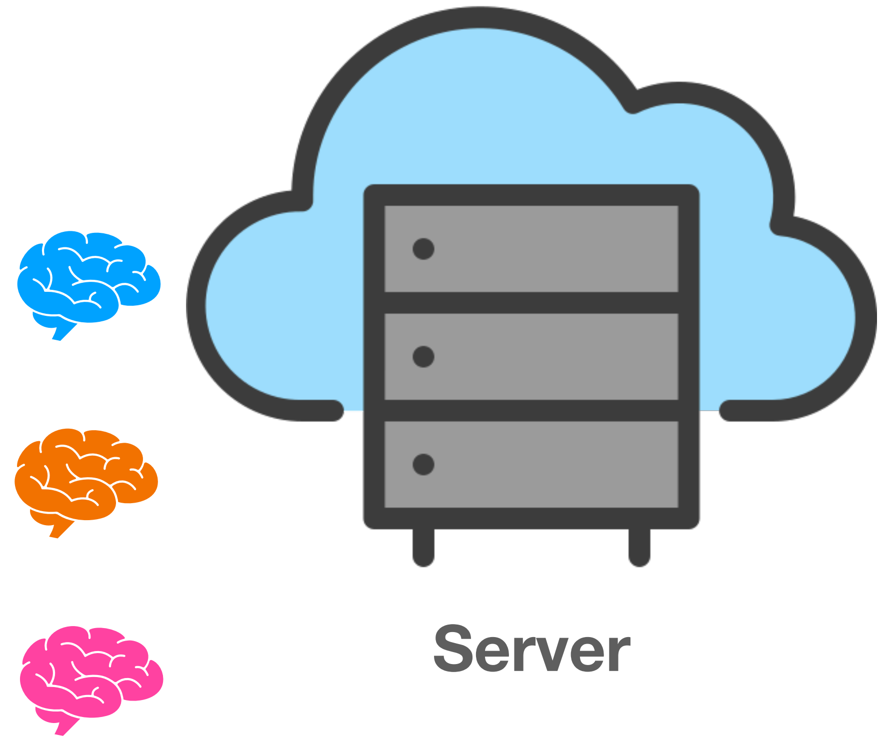

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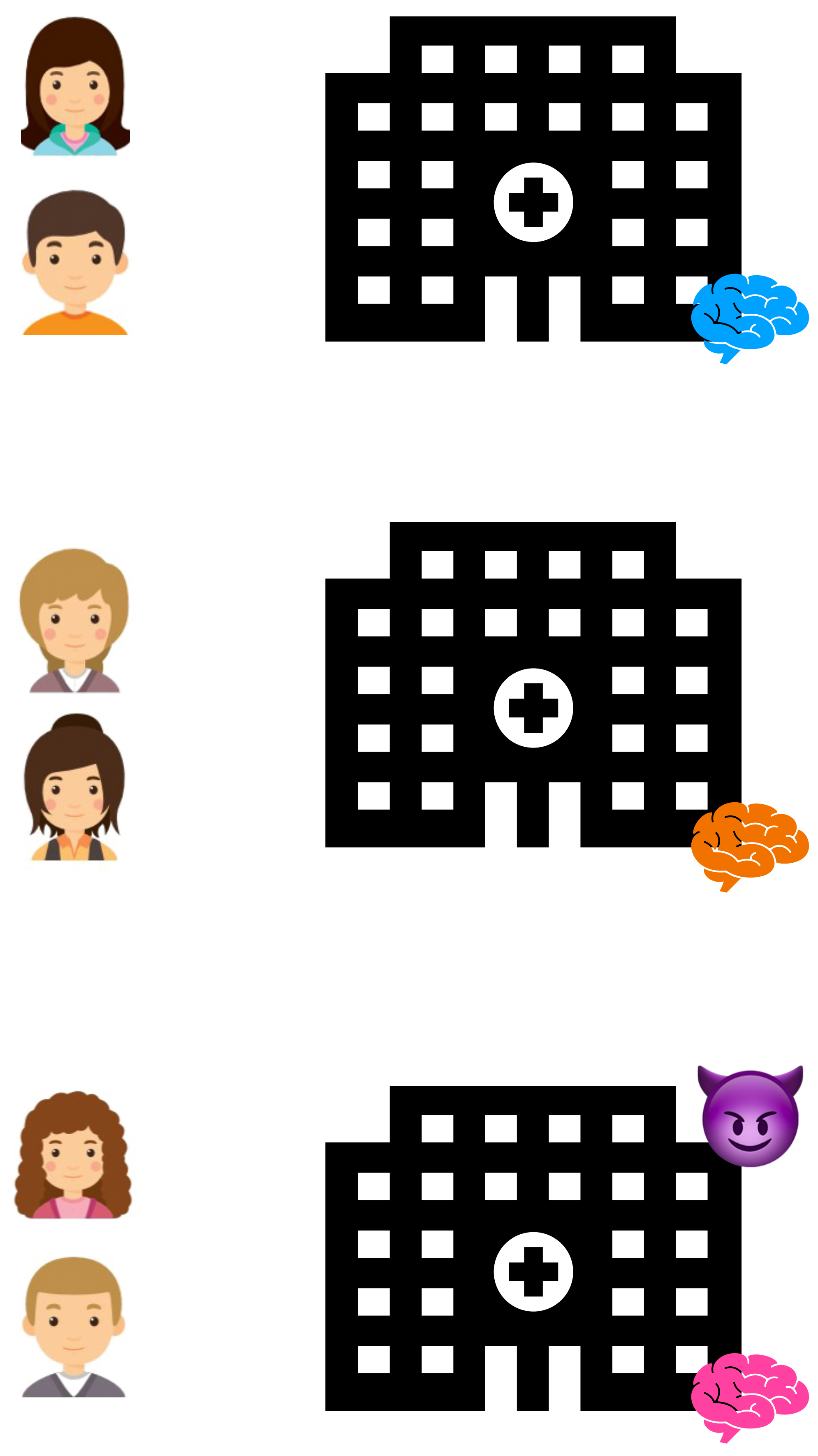

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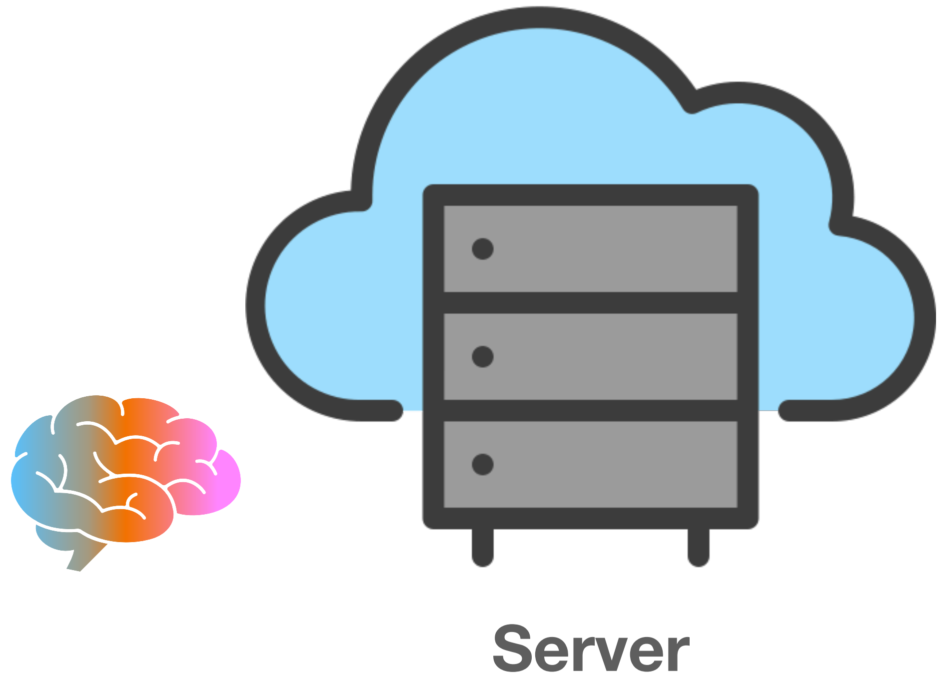

Malicious silo can take advantage of this to infer user's sensitive data in another silo

Privacy Gap in SOTA

A simple toy-example attack

Communication schedule for silos in SOTA

$$\exists i \in [M], \quad f(X_i, Z) > 0$$

- Z — all previous sync data among all silos
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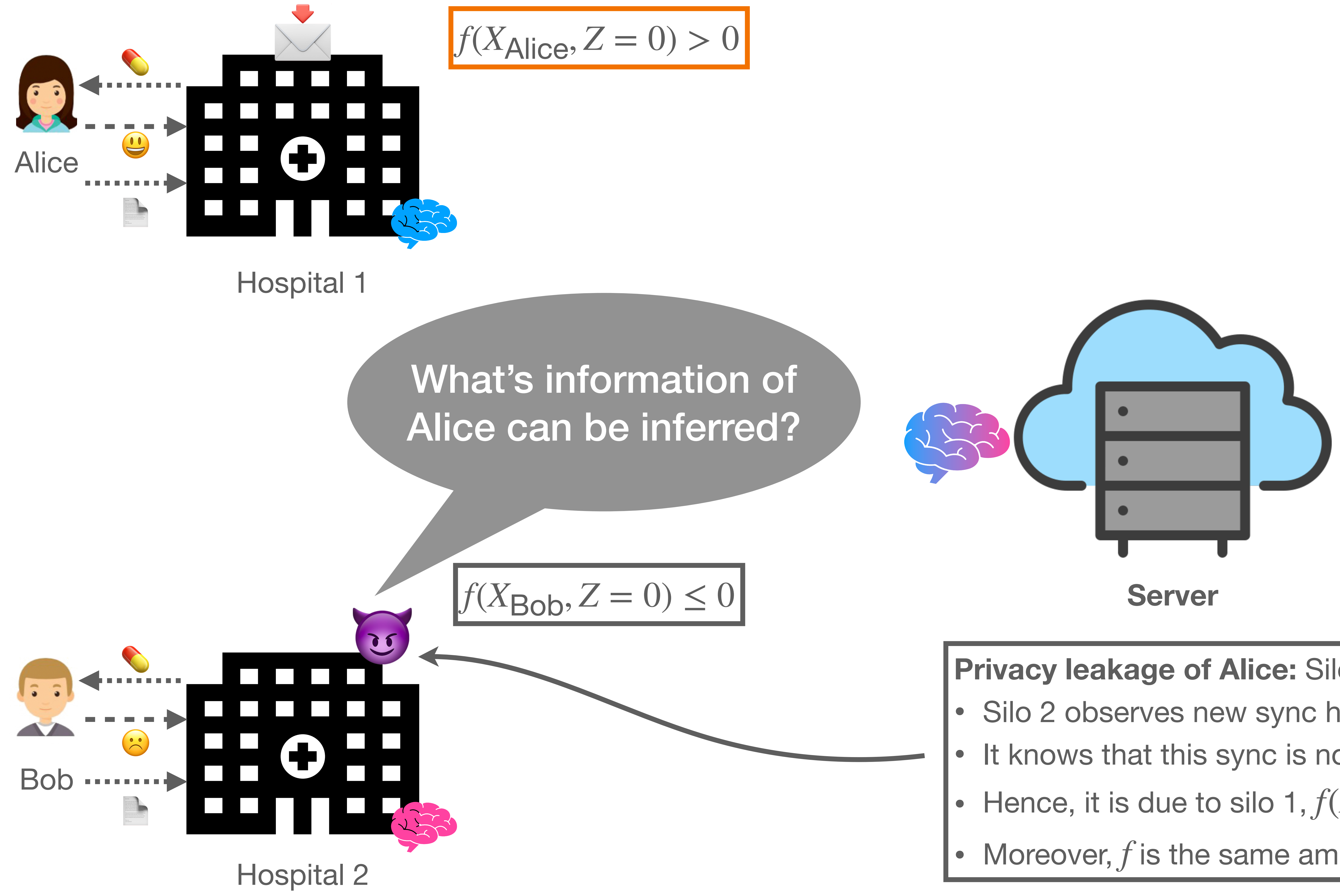


time $t = 1$

Privacy leakage of Alice: Silo 2 can infer Alice's data

- Silo 2 observes new sync happens
- It knows that this sync is not triggered by itself
- Hence, it is due to silo 1, $f(X_{\text{Alice}}, Z=0) > 0$
- Moreover, f is the same among silos

Privacy Gap in SOTA

A simple toy-example attack

Communication schedule for silos in SOTA

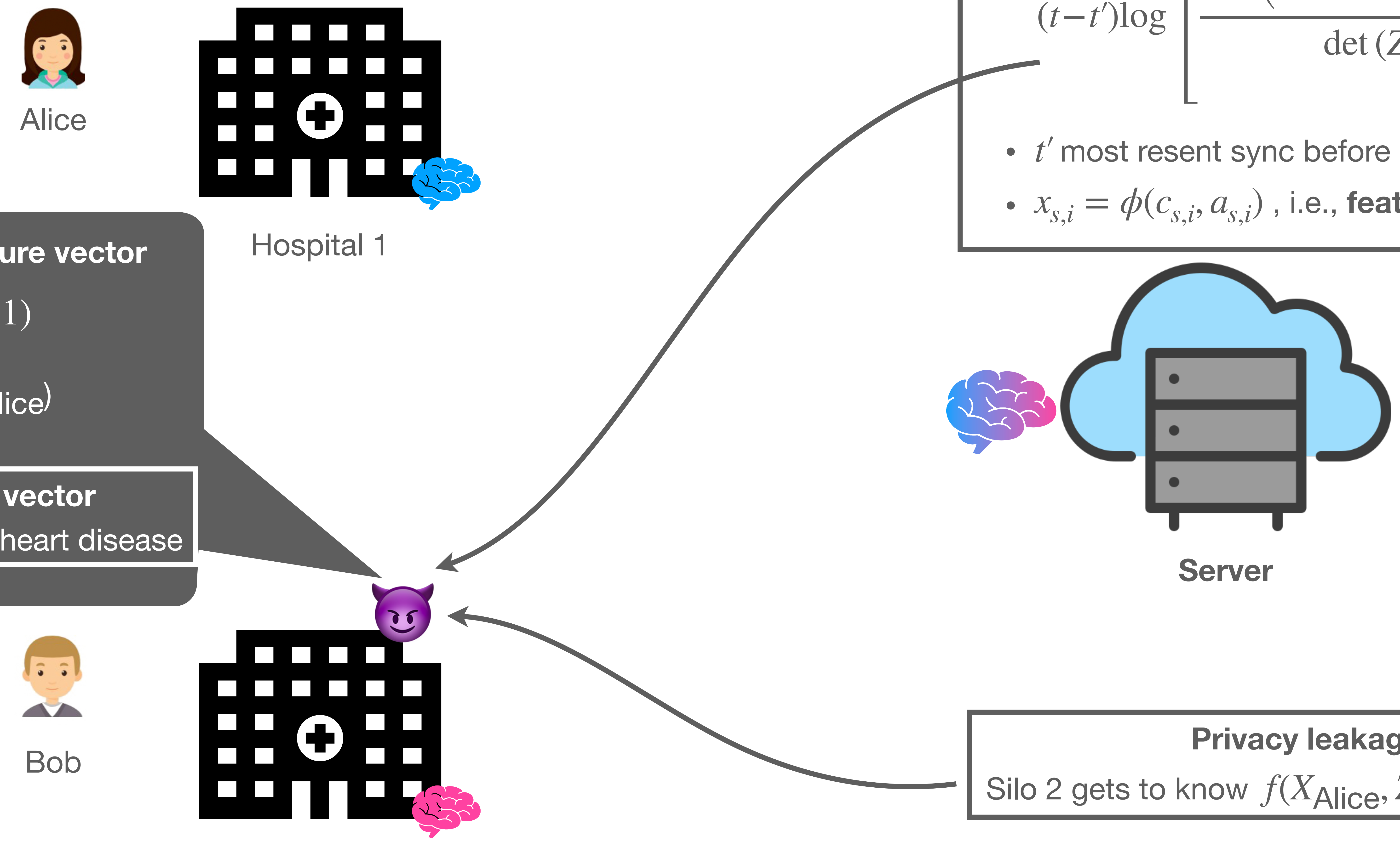
$$\exists i \in [M], \quad f(X_i, Z) > 0$$

- Z — all previous sync data among all silos
- X_i — **new non-private** local data at silo i since recent sync
- f — sync function, shared among all silos

In particular, a sync triggered by silo i at time t if

$$(t-t')\log\left[\frac{\det\left(Z+\sum_{s=t'+1}^{t} x_{s,i}x_{s,i}^{\top}+\lambda I\right)}{\det(Z+\lambda I)}\right] > D$$

- t' most resent sync before t and D some threshold
- $x_{s,i} = \phi(c_{s,i}, a_{s,i})$, i.e., **feature vector**



Silo 2 knows the norm of Alice's feature vector

$$\| x_{\text{Alice}} \|^2 > C := \lambda(e^D - 1)$$

$$(t = 1, t' = 0, Z = 0, x_{1,1} = x_{\text{Alice}})$$

Context info leaked via feature vector

i.e., Alice may have both diabetes and heart disease

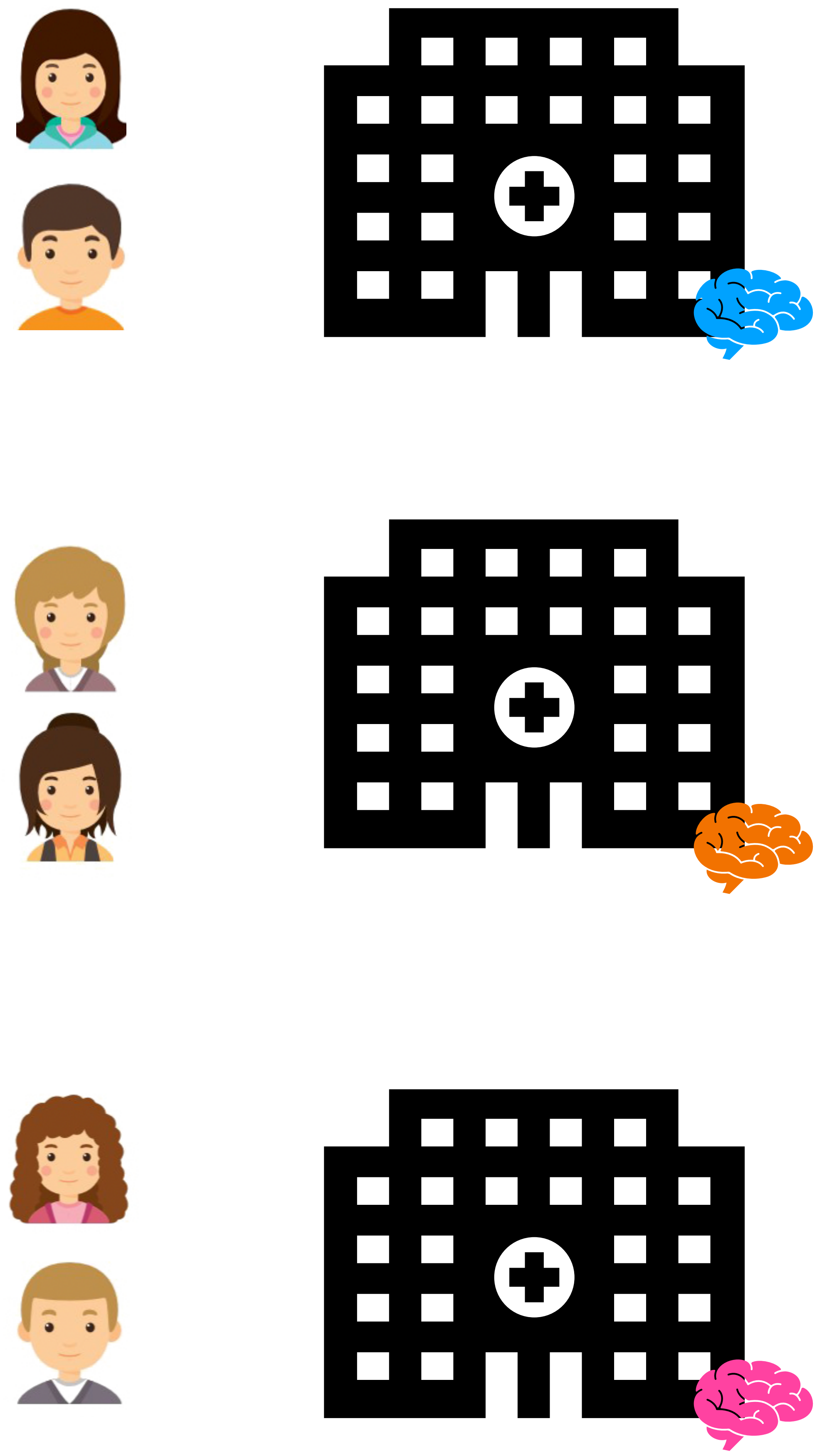
Privacy leakage of Alice

Silo 2 gets to know $f(X_{\text{Alice}}, Z = 0) > 0$ at time $t = 1$

Regret and Communication Gaps in SOTA

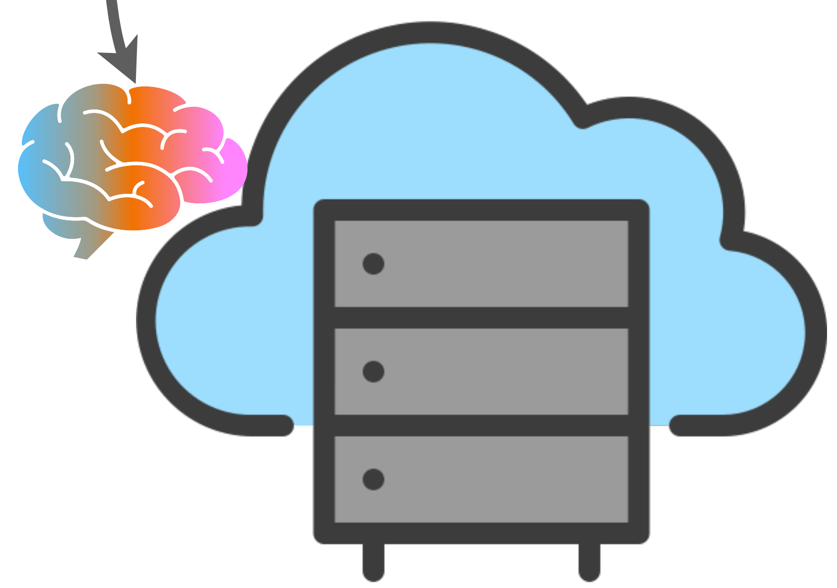
Low communication cost is also ungrounded

i.e., $O(\log T)$??



Larger total privacy noise implies larger regret

Ignore the privacy issue, the total amount of privacy noise in SOTA needs to be $\sigma^2_{\text{total}} = M\,\sigma^2$, i.e., M factor of its current one (recall M is the no. silos)



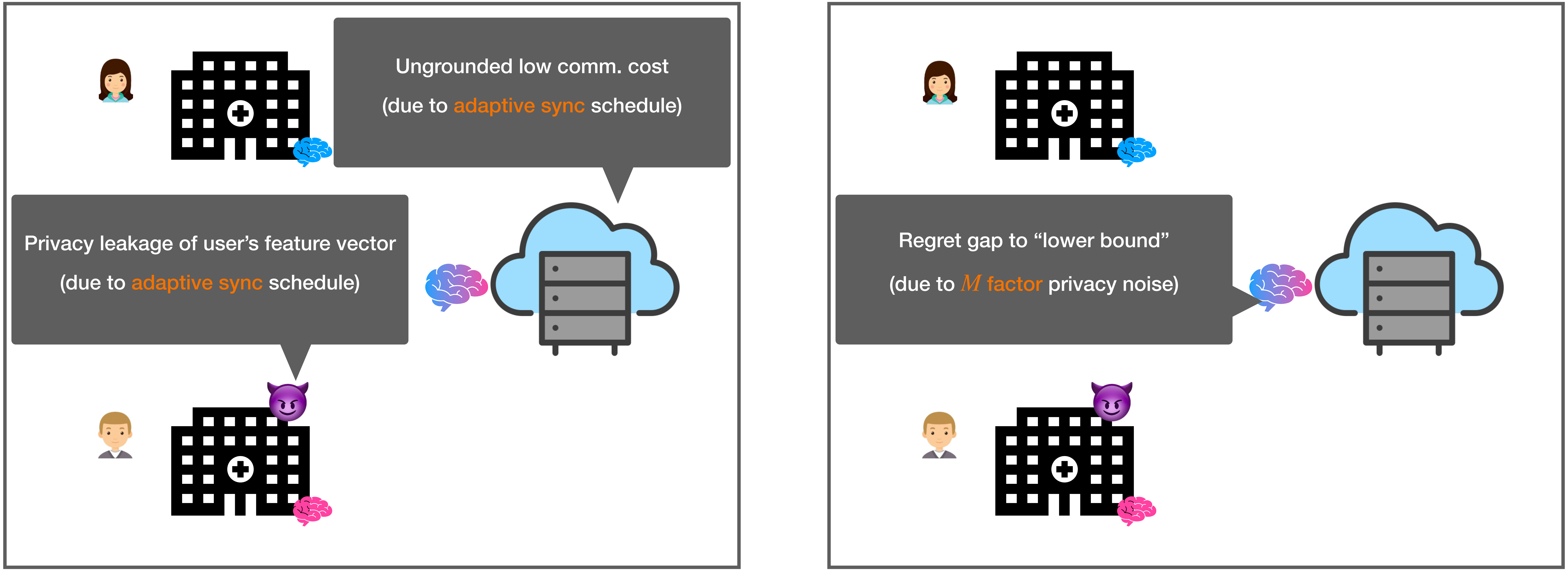
Current conclusion in SOTA becomes ungrounded

After the correction of M factor, the regret due to privacy changes from

$\tilde{O}(\sqrt{MT/\epsilon})$ *(match the "lower bound" of a single agent)*

to

$\tilde{O}(M^{3/4}\sqrt{T/\epsilon})$ *(has a gap of $M^{1/4}$ compared to "lower bound")*



Motivating Questions

1. How to address the privacy leakage? (💡 *a fixed communication schedule may work, i.e., does not depend on user's non-private data)*
2. How to correct the regret bound while preserving the privacy?
3. How to close the gap compared to the "lower bound" ? (💡 *need a way to get rid of M factor)*
4. **If possible, can we achieve all of them via a generic method? (**💡 *a template algorithm with a template proof is preferred***)**

Our Approach

A Generic Algorithm

Private-FedLinUCB

Private-FedLinUCB
(fixed batch sync of LinUCB with privacy)

Parameters: batch size B, privacy protocol $P = (R, S, A)$

Initialize: $\forall i, W_i = 0, U_i = 0$; $\widetilde{W}_{\text{syn}} = 0$, $\widetilde{U}_{\text{syn}} = 0$

for $t = 1, \ldots, T$ **do**

  for each agent $i = 1, \ldots, M$ **do**

    $V_{t,i} = \lambda I + \widetilde{W}_{\text{syn}} + W_i$, $U_{t,i} = \widetilde{U}_{\text{syn}} + U_i$

    Estimate: $\hat{\theta}_{t,i} = V_{t,i}^{-1} U_{t,i}$

    UCB: $a_{t,i} = \arg\max_a \phi(c_{t,i}, a)^\top \hat{\theta}_t + \beta_t \left\| \phi(c_{t,i}, a) \right\|_{V_{t,i}^{-1}}$

    Observe reward $y_{t,i}$; set $x_{t,i} = \phi(c_{t,i}, a_{t,i})$

    Update local data: $W_i = W_i + x_{t,i} x_{t,i}^\top$, $U_i = U_i + x_{t,i} y_{t,i}$

  if $t \bmod B = 0$ **then**

    $\widetilde{W}_{\text{syn}} = P(\{W_i\}_{i \in [M]})$, $\widetilde{U}_{\text{syn}} = P(\{U_i\}_{i \in [M]})$

    Receive $\widetilde{W}_{\text{syn}}$, $\widetilde{U}_{\text{syn}}$ from server

    Reset $W_i = 0, U_i = 0$

W_i — sum of local covariance matrices at agent i
U_i — sum of local bias vectors at agent i
$\widetilde{W}_{\text{syn}}$ — private sync covariance matrices among all agents
$\widetilde{U}_{\text{syn}}$ — private sync bias vectors among all agents

$V_{t,i}$ — sum of regularizer, sync and new local cov. matrices
$U_{t,i}$ — sum of sync and new local bias vectors

$P = (R, S, A)$, a template protocol for **summation** (will discuss it soon)
  R — local randomzier at agent side (on W_i, U_i)
  S — shuffler or identity mapping, between agents, server
  A — analyzer at server side

Single agent LinUCB[APS11] 101

For $t = 1, \ldots, T$:

1. **Estimate** θ^*: $\hat{\theta}_t = V_t^{-1} U_t$,

   $(V_t = \lambda I + \sum_{s=1}^{t-1} x_s x_s^\top$ ("covariance"), $U_t = \sum_{s=1}^{t-1} x_s y_s$ ("bias"))

2. **UCB:** $a_t = \arg\max_a \phi(c_t, a)^\top \hat{\theta}_t + \beta_t \left\| \phi(c_t, a) \right\|_{V_t^{-1}}$

   $(x_t = \phi(c_t, a_t)$, β_t — chosen via confidence bound)

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    Observe reward $y_{t,i}$; set $x_{t,i} = \phi(c_{t,i}, a_{t,i})$

    Update local data: $W_i = W_i + x_{t,i} x_{t,i}^\top$, $U_i = U_i + x_{t,i} y_{t,i}$

  if t mod $B = 0$ **then**

    $\widetilde{W}_{\text{syn}} = P(\{W_i\}_{i \in [M]})$, $\widetilde{U}_{\text{syn}} = P(\{U_i\}_{i \in [M]})$

    Receive $\widetilde{W}_{\text{syn}}$, $\widetilde{U}_{\text{syn}}$ from server

    Reset $W_i = 0, U_i = 0$

Single agent LinUCB[APS11] 101

For $t = 1,\ldots,T$:

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   $(x_t = \phi(c_t, a_t)$, β_t — chosen via confidence bound)

Remark: fixed vs. adaptive schedule

- It now suffices to privatize each sent messages for silo-level LDP guarantee
  - $R(W_i)$ and $R(U_i)$ is private at each sync round $k \in [T/B]$
  - without worrying privacy leakage via schedule
  - B needs to balance between comm. cost, regret, and privacy
- The problem in SOTA is: schedule depends on **non-private data** (i.e., W_i)
  - how about privatizing it first and then be adaptive?
  - we show that it will lead to fundamental challenge in regret analysis

$P = (R, S, A)$,a template protocol for **summation** (will discuss it soon)
R — local randomzier at agent side (on W_i, U_i)
S — shuffler or identity mapping, between agents, server
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A Generic Priv. Protocol

Distributed Tree-based alg.

P = (R,S,A), privacy protocol
(distributed version of Tree-based algorithm)

Differential Privacy 201

1. **Gaussian mechanism** for private sum of l_2 bounded vectors

   i.e., $\widetilde{s}$ is the private sum of $\sum_{s=1}^{k} \gamma_s$ under (ϵ, δ)-DP

   $$\widetilde{s} = \sum_{s=1}^{k} \gamma_s + \mathcal{N}(0, \sigma^2 I),\ \sigma^2 \approx \frac{L^2 \log(1/\delta)}{\epsilon^2}$$

   Intuition: change one data, the sum changes in l_2, bounded by L

2. **Continual private sum** (essential for private online learning)

   i.e., a stream of data $\gamma_1, \ldots, \gamma_K$, compute $\widetilde{s}_k$ — priv. sum of $\sum_{s=1}^{k} \gamma_s$

   Simple Approach I: add noise ($\approx 1/\epsilon^2$) to each γ_s

   — (ϵ, δ)-DP (by post-processing)

   — total noise is K/ϵ^2 (**!**)

   Simple Approach II: add noise ($\approx 1/\epsilon^2$) to each prefix sum

   — total noise is $1/\epsilon^2$ for all k

   — $\approx (\sqrt{K}\epsilon, \delta')$-DP (by composition of DP)

   — i.e., for (ϵ, δ)-DP, the total noise needs to be K/ϵ^2 (**!**)

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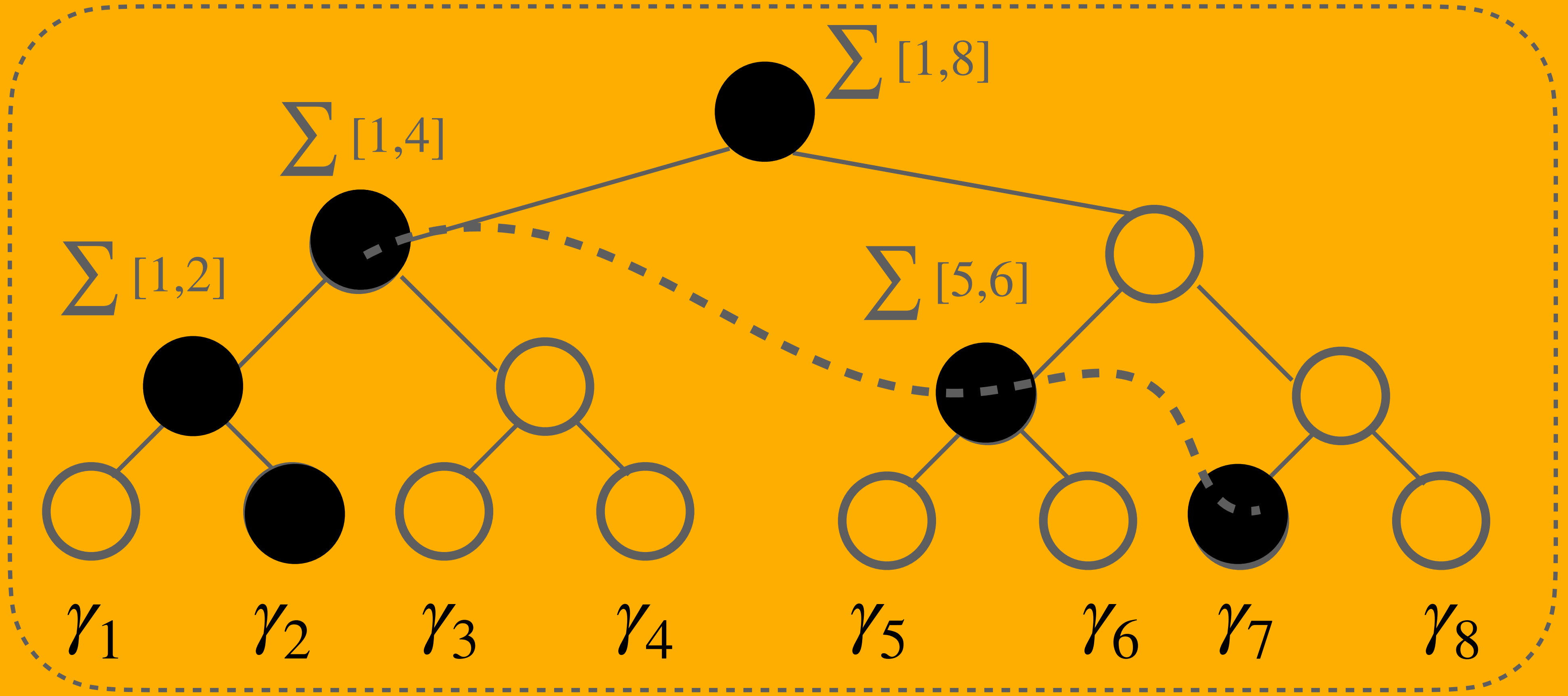
   i.e., a stream of data $\gamma_1, \ldots, \gamma_K$, compute $\widetilde{s}_t$ — priv. sum of $\sum_{s=1}^{k} \gamma_s$

   Tree-based algorithm [CSS11]: add noise to partial sum $\sum [i, j]$



Key observations:

— each data affects at most $O(\log K)$ p-sums ($\widetilde{O}(1/\epsilon^2)$ noise each)

— each prefix sum needs at most $O(\log K)$ partial-sums (p-sums)

— total noise is still $\widetilde{O}(1/\epsilon^2)$ (✅ ignore log factor)

A Generic Priv. Protocol

Distributed Tree-based alg.

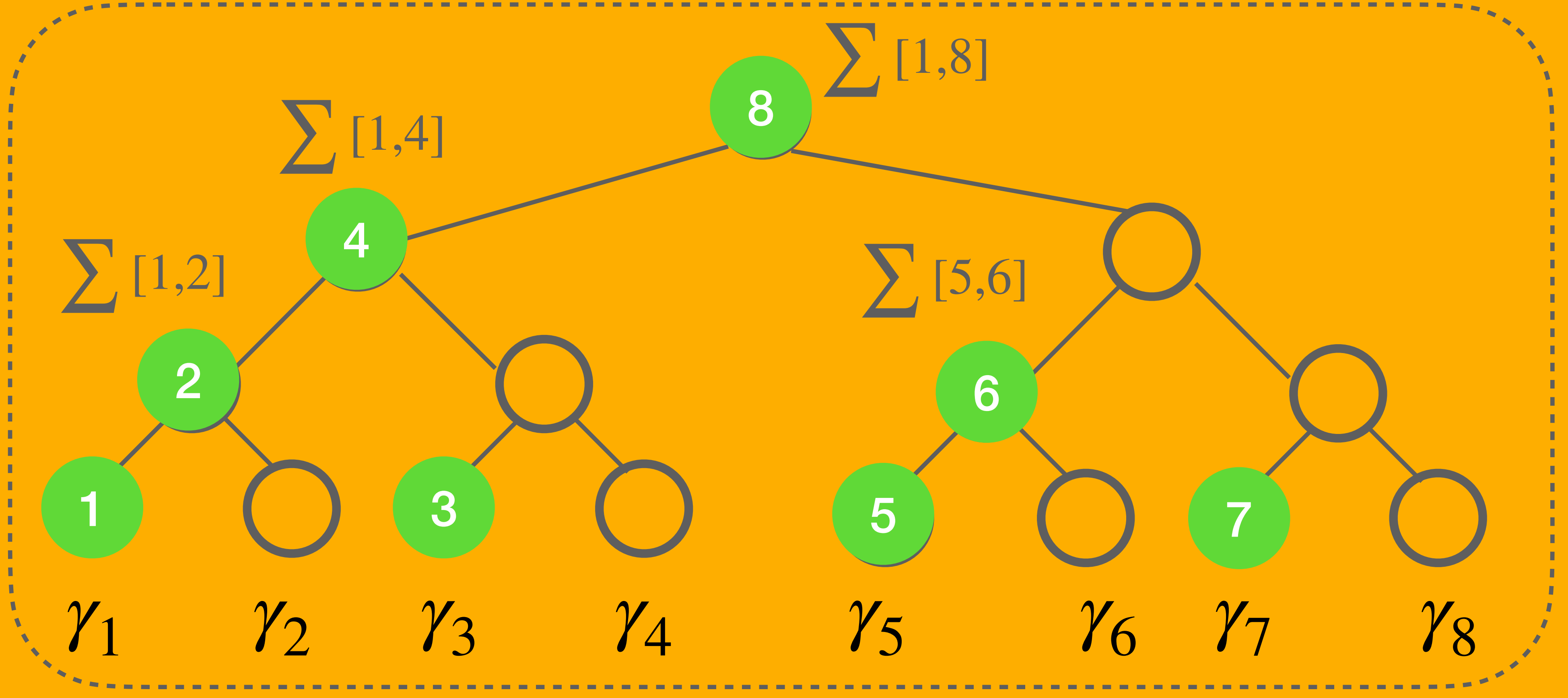
`P = (R,S,A), privacy protocol`

(distributed version of Tree-based algorithm)

Differential Privacy 201

Tree-based algorithm [CSS11] for continual private sum

i.e., a stream of data $\gamma_1, \ldots, \gamma_K$, compute $\tilde{s}_t$ — priv. sum of $\sum_{s=1}^{k} \gamma_s$



Sequential implementation (dynamically compute p-sum, tree node)

1. For each round k, express k in binary form: $k = \sum_j \text{Bin}_j(k) \cdot 2^j$
   (e.g., for $k = 6$, it is 110)
2. Find index of first one $i_k = \min\{j : \text{Bin}_j(k) = 1\}$ (for $k = 6$, $i_k = 1$)
3. Compute non-private p-sum: $\alpha_{i_k} = \sum_{j<i_k} \alpha_j + \gamma_k$
   (α_j — stores the sum of 2^j data)
4. Private p-sum $\tilde{\alpha}_{i_k} = \alpha_{i_k} + \mathcal{N}(0, \sigma^2 I)$
5. Final output $\tilde{s}_k = \sum_{j:\text{Bin(k)}=1} \tilde{\alpha}_j$ (for $k = 6$, [1,4] + [5,6])

A Generic Priv. Protocol

Distributed Tree-based alg.

P = (R,S,A), privacy protocol
(distributed version of Tree-based algorithm)

Procedure: Local Randomizer R at each agent $i \in [M]$

for each sync $k = 1,\ldots,K$ **do**

Express k in binary form: $k = \sum_j \text{Bin}_j(k) \cdot 2^j$

Find index of first one $i_k = \min\{j : \text{Bin}_j(k) = 1\}$

Compute non-private p-sum: $\alpha_{i_k} = \sum_{j<i_k} \alpha_j + \gamma_k$

Output noisy p-sum $\widetilde{\alpha}_{i_k,i} = \alpha_{i_k} + \mathcal{N}(0,\sigma^2 I)$

Procedure: Shuffler S (could be empty or identity mapping)

Procedure: Analyzer A at server

for each sync $k = 1,\ldots,K$ **do**

Express k in binary form and find index of first one i_k

Add noisy p-sums from **all agents** $\widetilde{\alpha}_{i_k} = \sum_{i\in[M]} \widetilde{\alpha}_{i_k,i}$

Output total sum: $\widetilde{s}_k = \sum_{j:\text{Bin(k)}=1} \widetilde{\alpha}_j$

Differential Privacy 201

Tree-based algorithm [CSS11] for continual private sum

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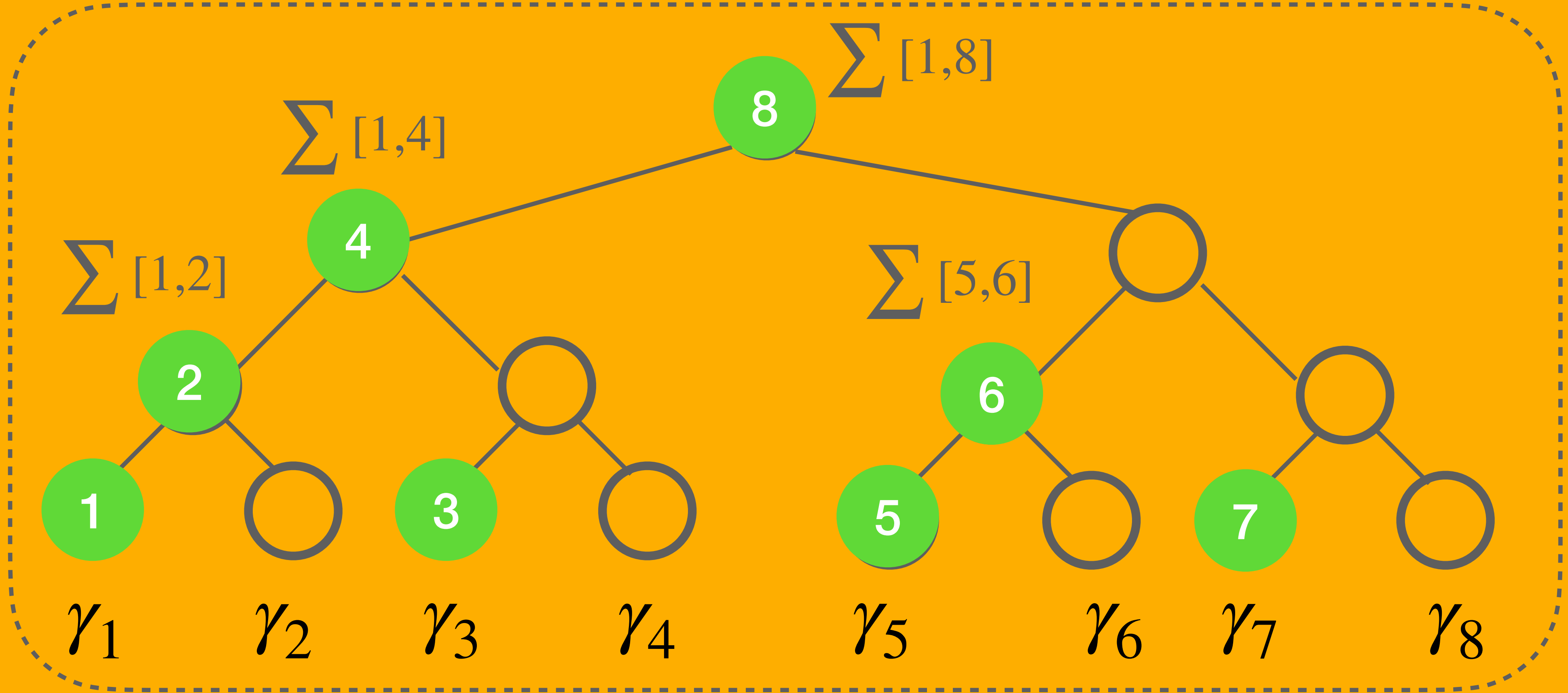

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Private-FedLinUCB

(fixed batch sync of LinUCB with privacy)

Parameters: batch size B, privacy protocol $P = (R, S, A)$

Initialize: $\forall i, W_i = 0, U_i = 0$; $\widetilde{W}_{\text{syn}} = 0$, $\widetilde{U}_{\text{syn}} = 0$

for $t = 1, \ldots, T$ **do**

for each agent $i = 1, \ldots, M$ **do**

$V_{t,i} = \lambda I + \widetilde{W}_{\text{syn}} + W_i$, $U_{t,i} = \widetilde{U}_{\text{syn}} + U_i$

Estimate: $\hat{\theta}_{t,i} = V_{t,i}^{-1} U_{t,i}$

UCB: $a_{t,i} = \arg\max_a \phi(c_{t,i}, a)^\top \hat{\theta}_t + \beta_t \left\| \phi(c_{t,i}, a) \right\|_{V_{t,i}^{-1}}$

Observe reward $y_{t,i}$; set $x_{t,i} = \phi(c_{t,i}, a_{t,i})$

Update local data: $W_i = W_i + x_{t,i} x_{t,i}^\top$, $U_i = U_i + x_{t,i} y_{t,i}$

if $t \bmod B = 0$ **then**

$\widetilde{W}_{\text{syn}} = P(\{W_i\}_{i \in [M]})$, $\widetilde{U}_{\text{syn}} = P(\{U_i\}_{i \in [M]})$

Receive $\widetilde{W}_{\text{syn}}$, $\widetilde{U}_{\text{syn}}$ from server

Reset $W_i = 0, U_i = 0$

$$\gamma_k^{\text{cov}} = \sum_{t=(k-1)B+1}^{kB} x_t x_t^\top$$

$$\gamma_k^{\text{bias}} = \sum_{t=(k-1)B+1}^{kB} x_t y_t$$

P = (R,S,A), privacy protocol

(distributed version of Tree-based algorithm)

Procedure: Local Randomizer R at each agent $i \in [M]$

for each sync $k = 1, \ldots, K$ **do**

Express k in binary form: $k = \sum_j \text{Bin}_j(k) \cdot 2^j$

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Express k in binary form and find index of first one i_k

Add noisy p-sums from **all agents** $\widetilde{\alpha}_{i_k} = \sum_{i \in [M]} \widetilde{\alpha}_{i_k, i}$

Output total sum: $\widetilde{s}_k = \sum_{j : \text{Bin(k)} = 1} \widetilde{\alpha}_j$

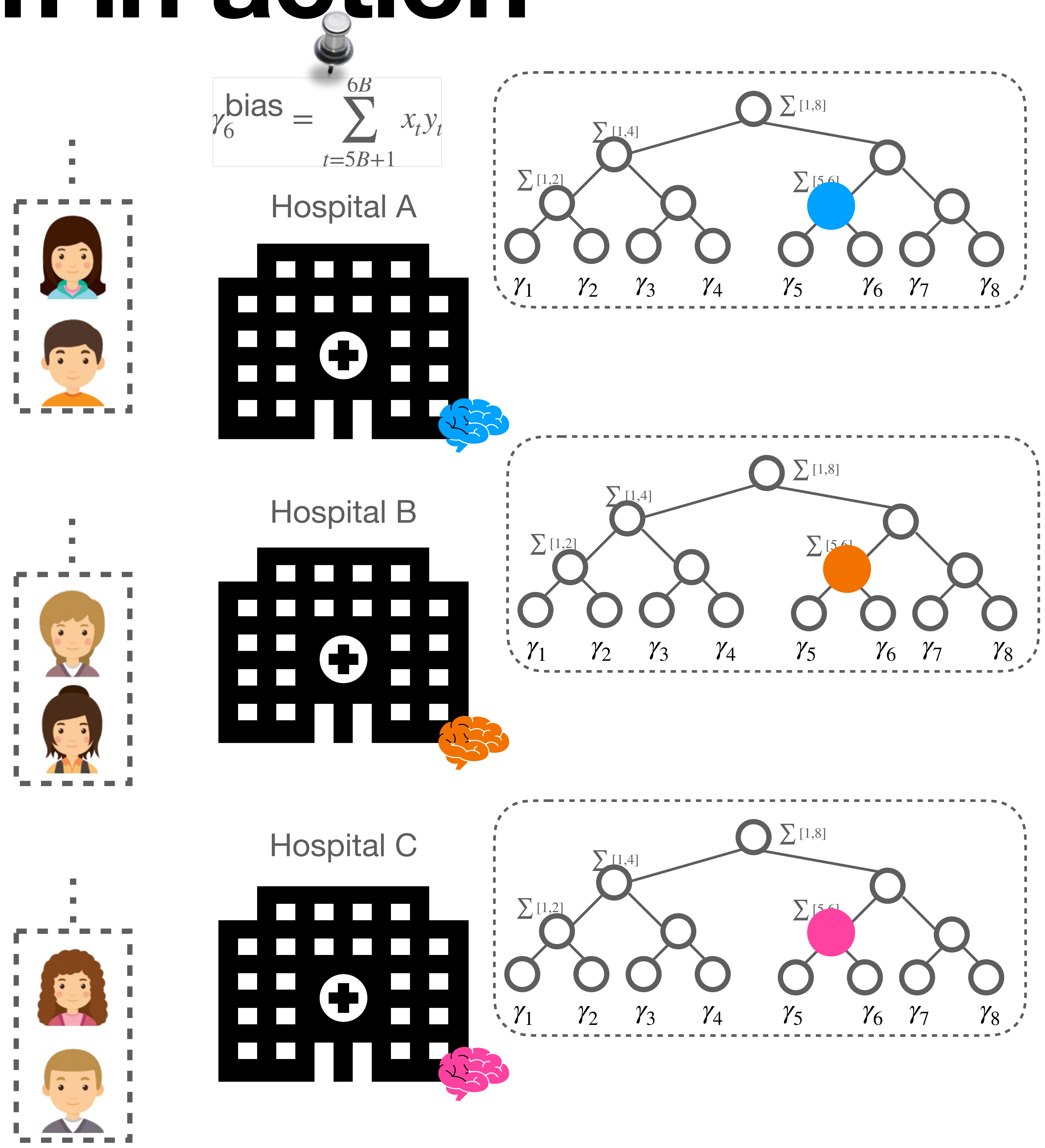
Putting them together

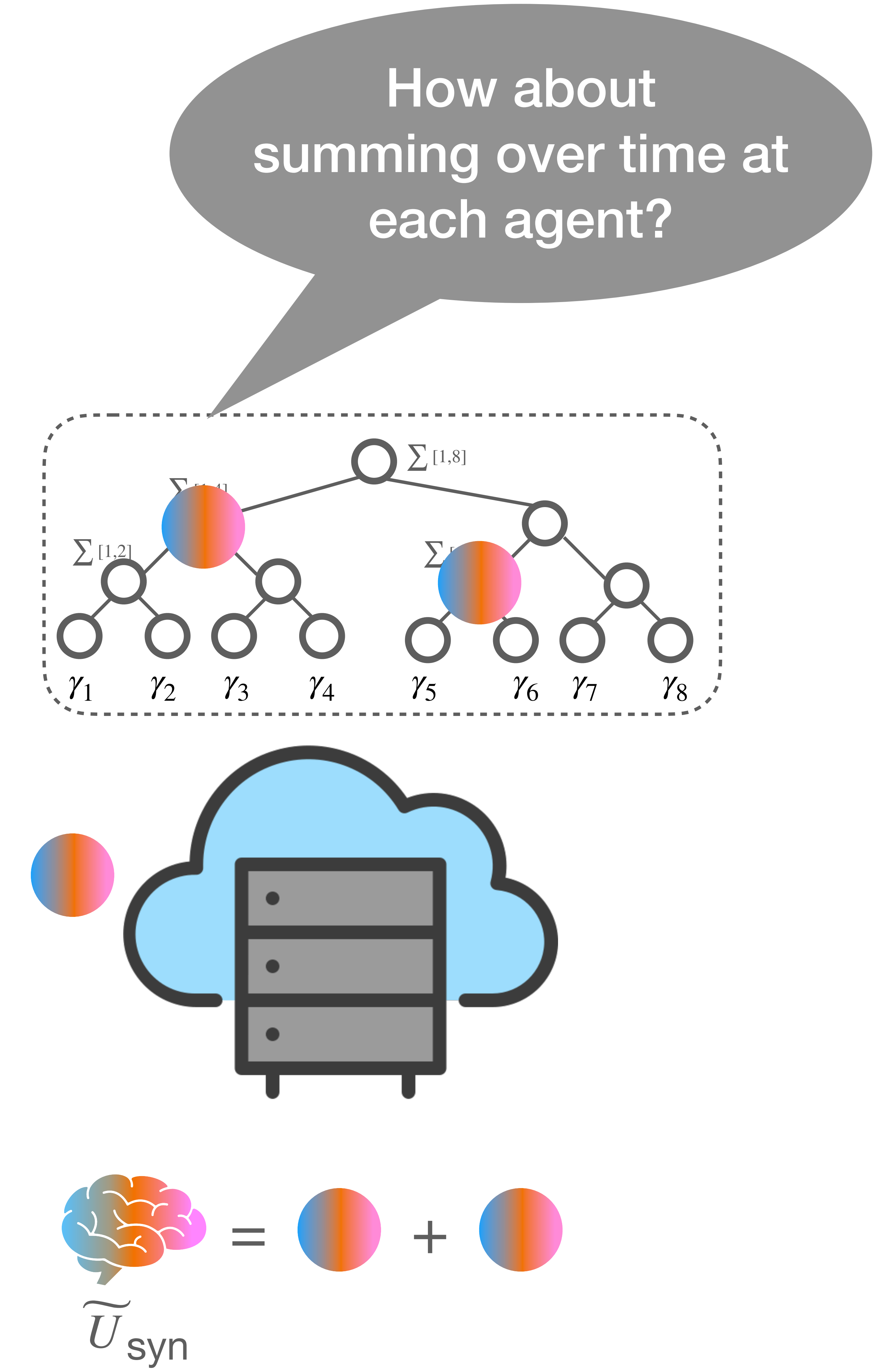
- Each agent runs two privacy protocol — sum of **covariance** matrices (i.e., W_i) and sum of **bias** vectors (i.e., U_i)
- The datapoint γ_k is a **batch** of data — total matrices or vectors during the kth batch
- **The sensitivity does not scale with respect to B**

Algorithm in action

Illustration

time $t = 6B$



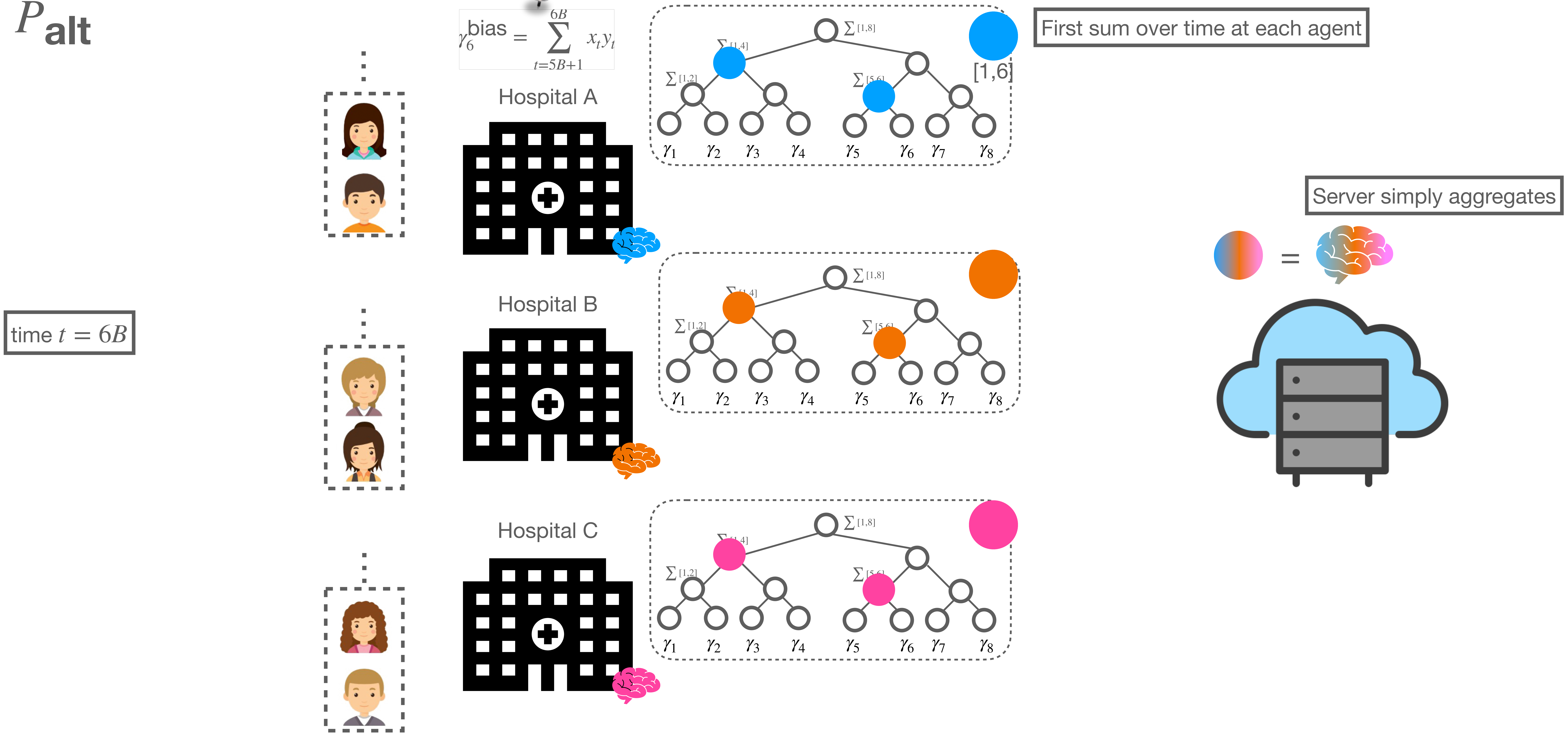


Private sum across both time and agents

$$\sum_{i=1}^{M} \sum_{t=1}^{6B} x_{t,i} y_{t,i}$$

An alternative protocol

P_{alt}



An alternative protocol

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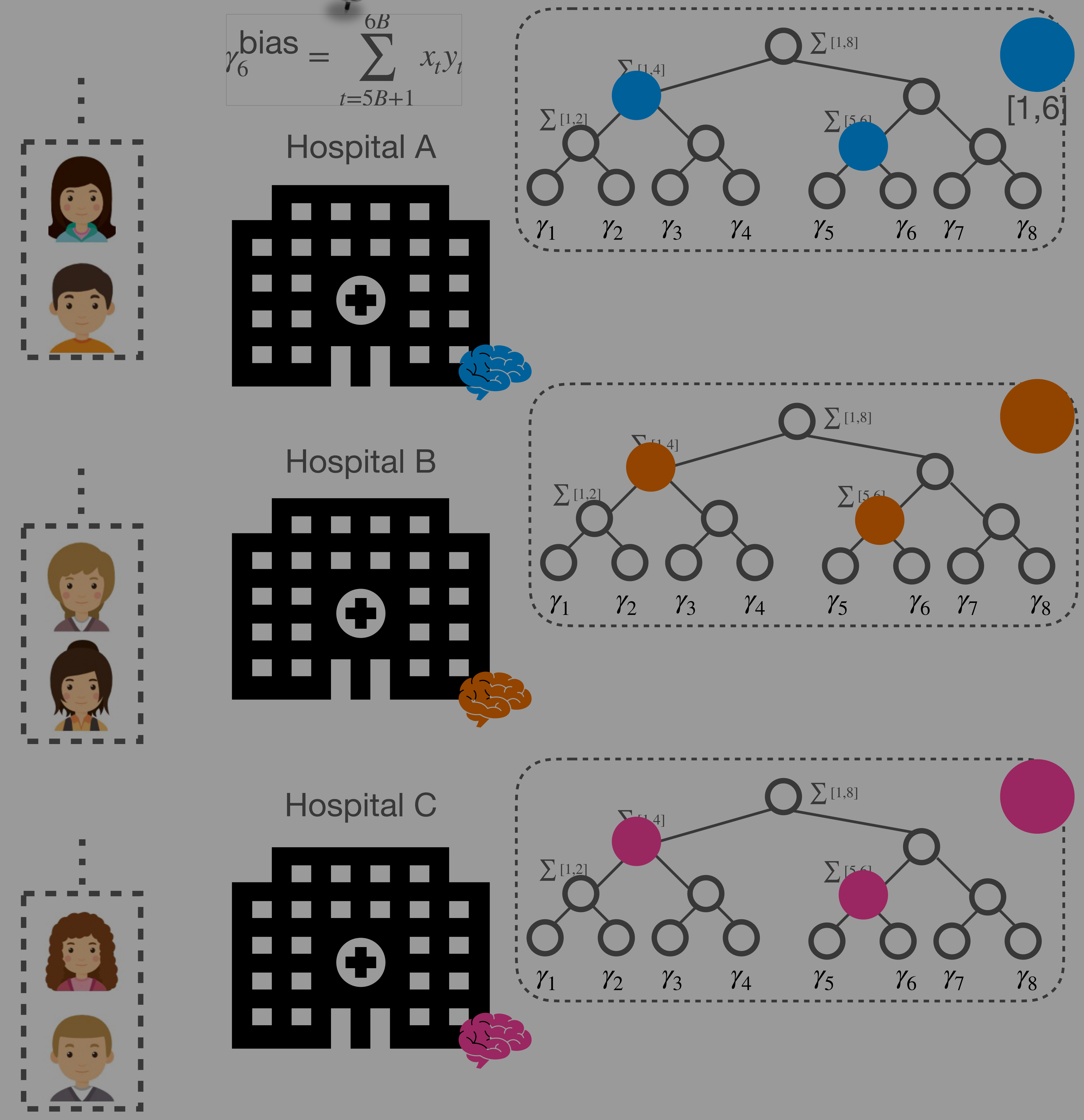
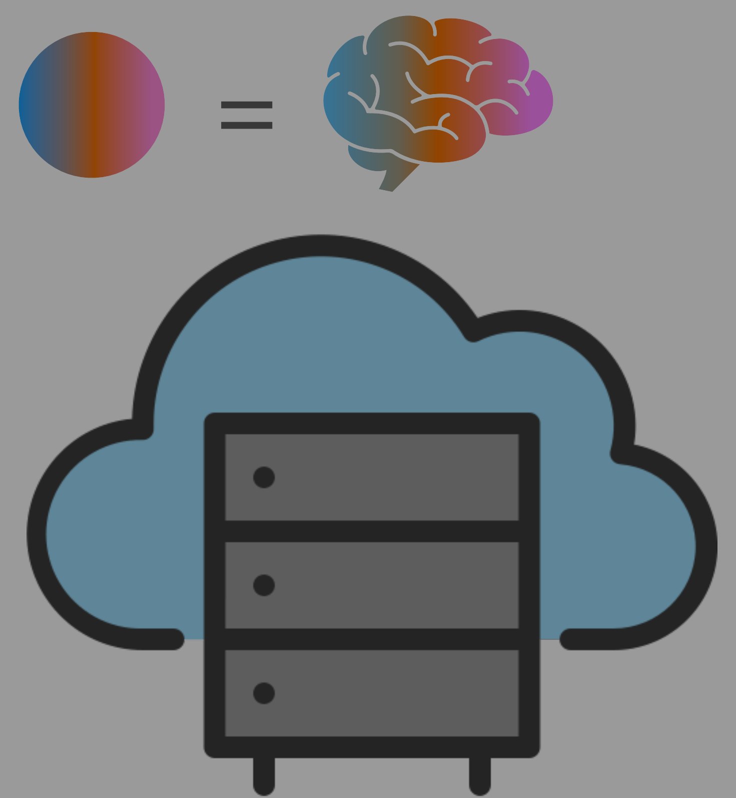


First sum over time at each agent

Server simply aggregates



Remark: comparisons

- As we will see, both protocols work for silo-level LDP
  - same regret under same (ϵ, δ)-DP
- However, for shuffle DP, things are different
  - our protocol manages to close the gap
  - P_{alt} fails to close the gap (more on this later…😉)

Theoretical Results

Federated LCBs under Silo-level LDP

Fix the issues in SOTA

Theorem 1 (Performance under silo-level LDP, informal)

Let batch size $B = \sqrt{T/M}$, privacy noise in P be $\sigma^2 = 8\kappa \cdot \dfrac{\log(2/\delta) + \epsilon}{\epsilon^2}$ with $\kappa = 1 + \log(T/B)$. Then, `Private-FedLinUCB` enjoys

1. **Privacy** — (ϵ, δ)-silo-level LDP for any $\epsilon > 0,\ \delta \in (0,1)$
2. **Regret** — $R_M(T) =$ non-private regret $+ \sqrt{T}\dfrac{(Md)^{3/4}\log^{1/4}(1/\delta)}{\sqrt{\epsilon}}$
3. **Communication** — $\sqrt{MT}$ rounds of sync between agents and server

Remark: comparisons with related work

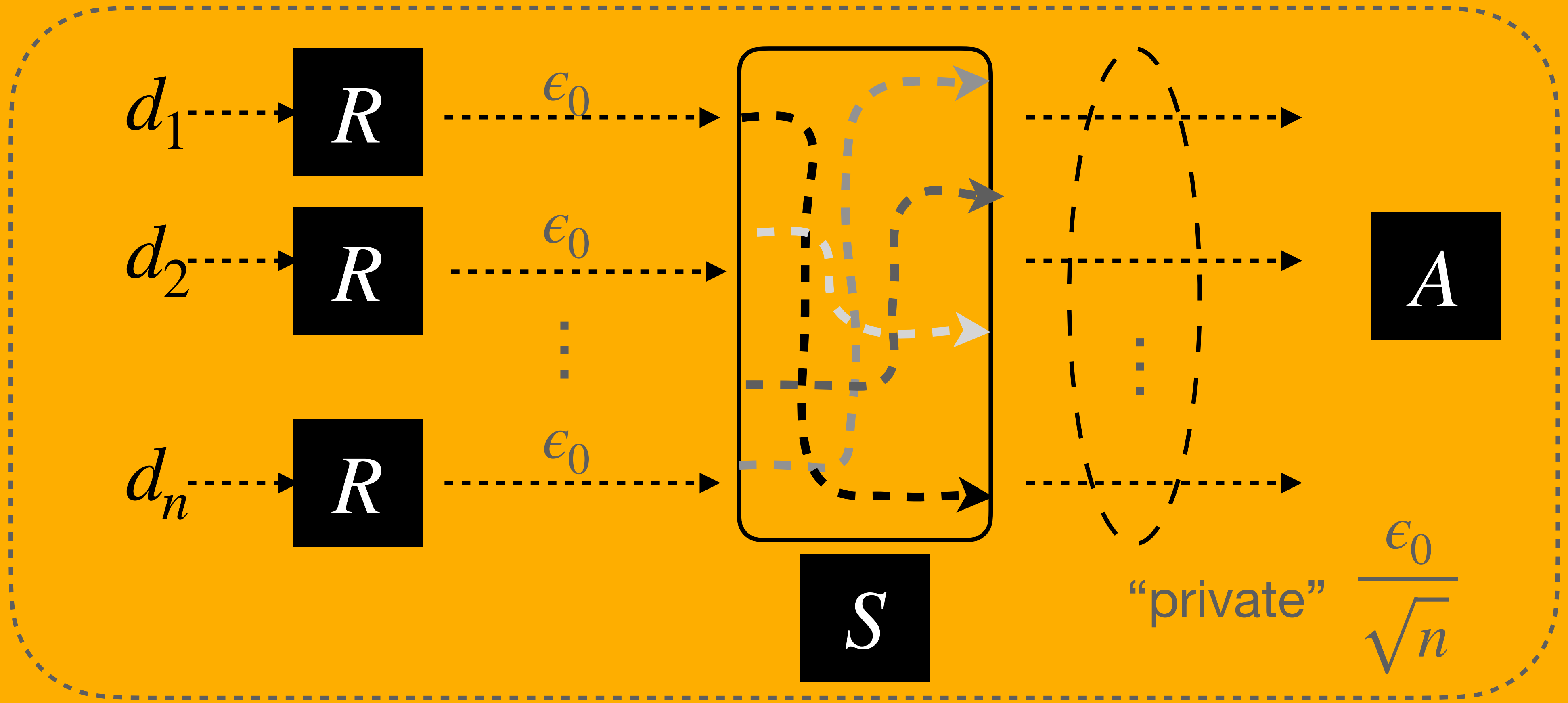
1. **Compared with SOTA[DP20]**
   — privacy: we fix the privacy leakage, thanks to the fixed-batch schedule and tree-based algorithm
   — regret: we establish the correct privacy cost, i.e., the additional regret due to privacy now scales with $M^{3/4}$ (instead of $\sqrt{M}$)
   — communication: communication is worse than SOTA ($\sqrt{T} vs . \log T$) due to fixed-batch comm. But, note that there exists privacy leakage
2. **Compared with "super" single agent under central DP[SS18]**
   — our regret is $M^{1/4}$ factor worse than this "lower bound"

Federated LCBs under SDP

Match the “lower bound”

Differential Privacy 501

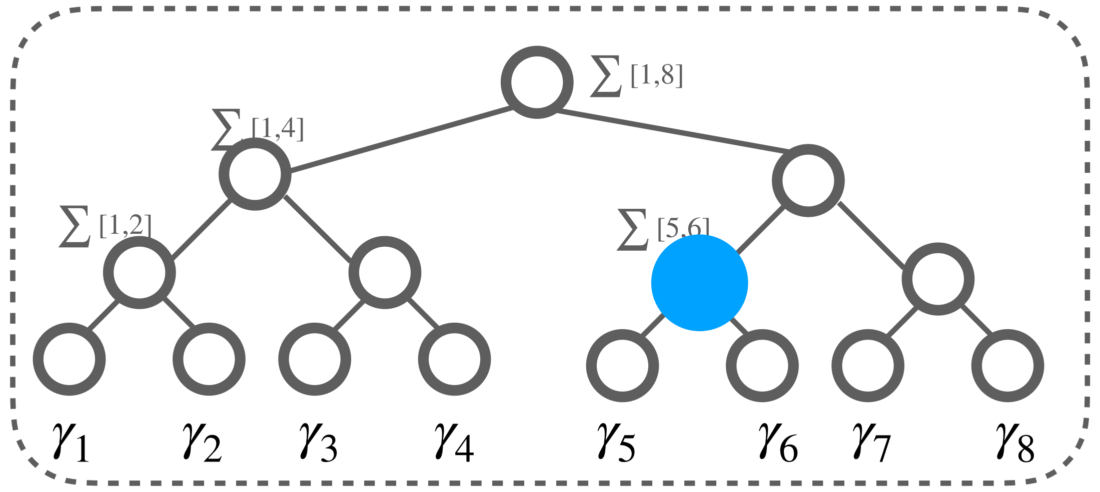
1. **What is shuffle DP (SDP)?**
   — formally defined in [CSUZZ19]
   — $P = (R, S, A)$, “the output of shuffler is private”
   — (change any d_i, the outputs are “close”)



2. **How to achieve it?**
   — one way is via LDP amplification, e.g., [FMT20]
   — shuffle n LDP outputs (each ϵ_0-DP), then it is $\approx \epsilon_0/\sqrt{n}$ SDP
   — **“reduce the privacy loss by a factor of $1/\sqrt{n}$ ✅”**
   — (intuition: hiding among clones)

Federated LCBs under SDP

Match the "lower bound"



How about adding shuffler between agents and server? $1/\sqrt{M}$

Good news: this amplification can close the gap ✅

Bad news: one cannot directly use existing results ❗

— they only amplify LDP (R oper. on single data)

— in our case, R oper. on multiple datapoints

— (this leads to key difference in the analysis)

Clones are harder to create due to multiple local points

A new amplification lemma is derived ✅

— tailored for Gaussian DP mecha.

— avoid group privacy

— control the blow up in δ

Differential Privacy 501

1. **What is shuffle DP (SDP)?**
   — formally defined in [CSUZZ19]
   — $P = (R, S, A)$, "the output of shuffler is private"
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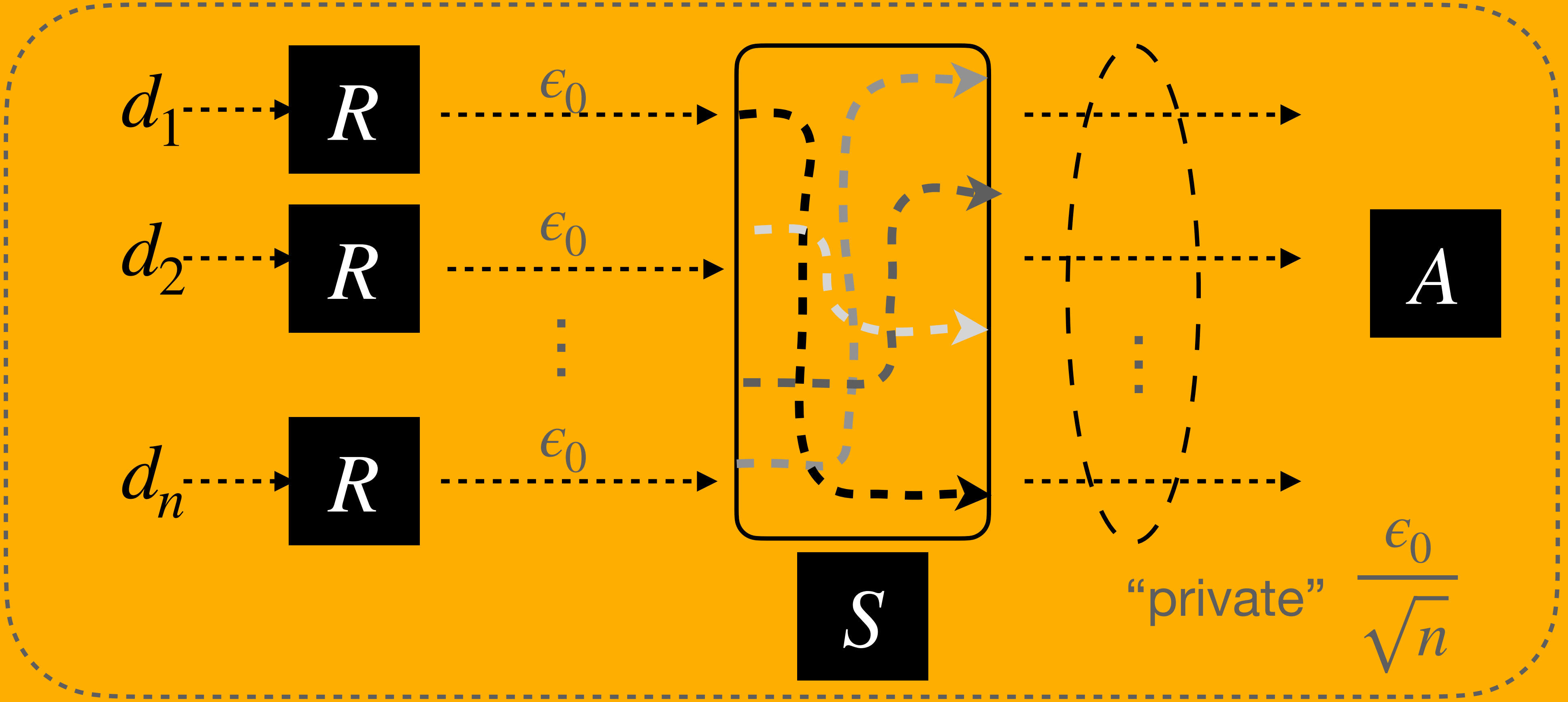

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Federated LCBs under SDP

Match the “lower bound”

How to improve the privacy guarantees?

Theorem 2 (Performance under SDP, informal)

Let batch size $B = \sqrt{T/M}$ and $\kappa = 1 + \log(T/B)$, privacy noise in P be $\sigma^2 = \tilde{O}\left(\frac{\kappa \log(1/\delta)}{\epsilon^2 M}\right)$. Then, `Private-FedLinUCB (with shuffler)` enjoys

1. **Privacy** — (ϵ, δ)-SDP for any $\epsilon \in \left(0, \frac{\sqrt{\kappa}}{C_1 T \sqrt{M}}\right)$, $\delta \in \left(0, \frac{\kappa}{C_2 T}\right)$, where C_1, C_2 are constants
2. **Regret** — $R_M(T) =$ non-private regret $+ \sqrt{MT}\frac{d^{3/4}\log^{3/4}(M\kappa/\delta)}{\sqrt{\epsilon}}$
3. **Communication** — $\sqrt{MT}$ rounds of sync between agents and server

Match the “lower bound”
Privacy cost is on the order of $\sqrt{MT}$

Privacy holds for small ϵ only
This comes from two factors due to amplification lemma
— $1/\sqrt{M}$ is the standard term
— $1/T$ is the new term due to multiple local points

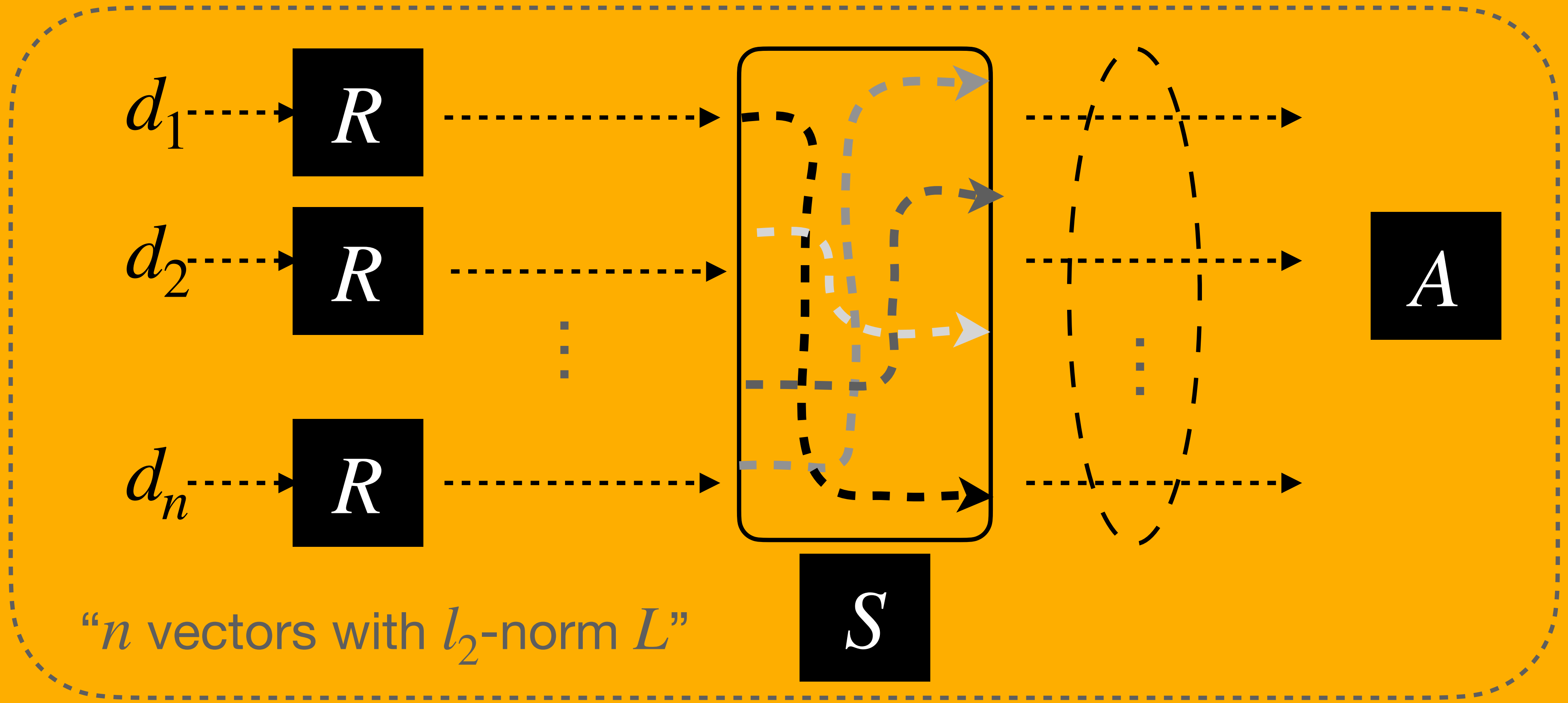
Minimum modifications
Compared to silo-level LDP, one only needs to
— add a shuffler
— adjust the noise in local randomizer R

Federated LCBs under SDP

Leverage vector-sum protocol

Differential Privacy 502

1. **How to achieve SDP?**
   — instead of using amplification lemma
   — one can use specific shuffle protocol
   — $P_{\text{vec}} = (R_{\text{vec}}, S, A_{\text{vec}})$[CJMP21] is one example

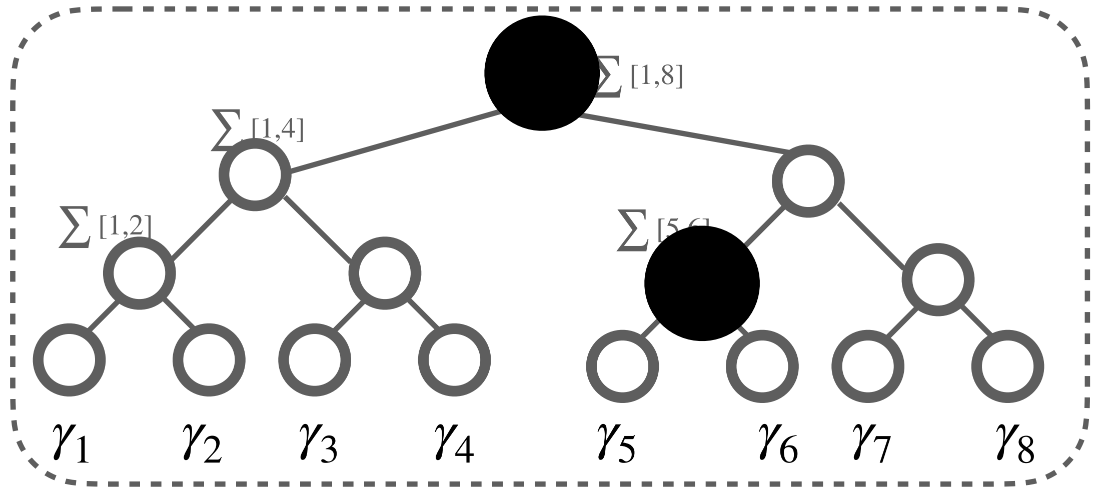


2. **Performance of P_{vec}**
   — it guarantees SDP for all $\epsilon \in (0,15)$, $\delta \in (0,1/2)$
   — the injected noise is $\frac{L^2}{\epsilon^2} \log^2(d/\delta)$ per entry **(indep. of n)**

(Essentially, it simulates central model without a trusted server)

Federated LCBs under SDP

Leverage vector-sum protocol



Can we simply use P_{vec} to add all p-sums across M agents?

The norm of p-sum could be linear with T !

— sum of M p-sums with P_{vec} (i.e., $n = M$)

— each data point has a large norm

View n in P_{vec} as data points across agents

—e.g., for $k = 6$

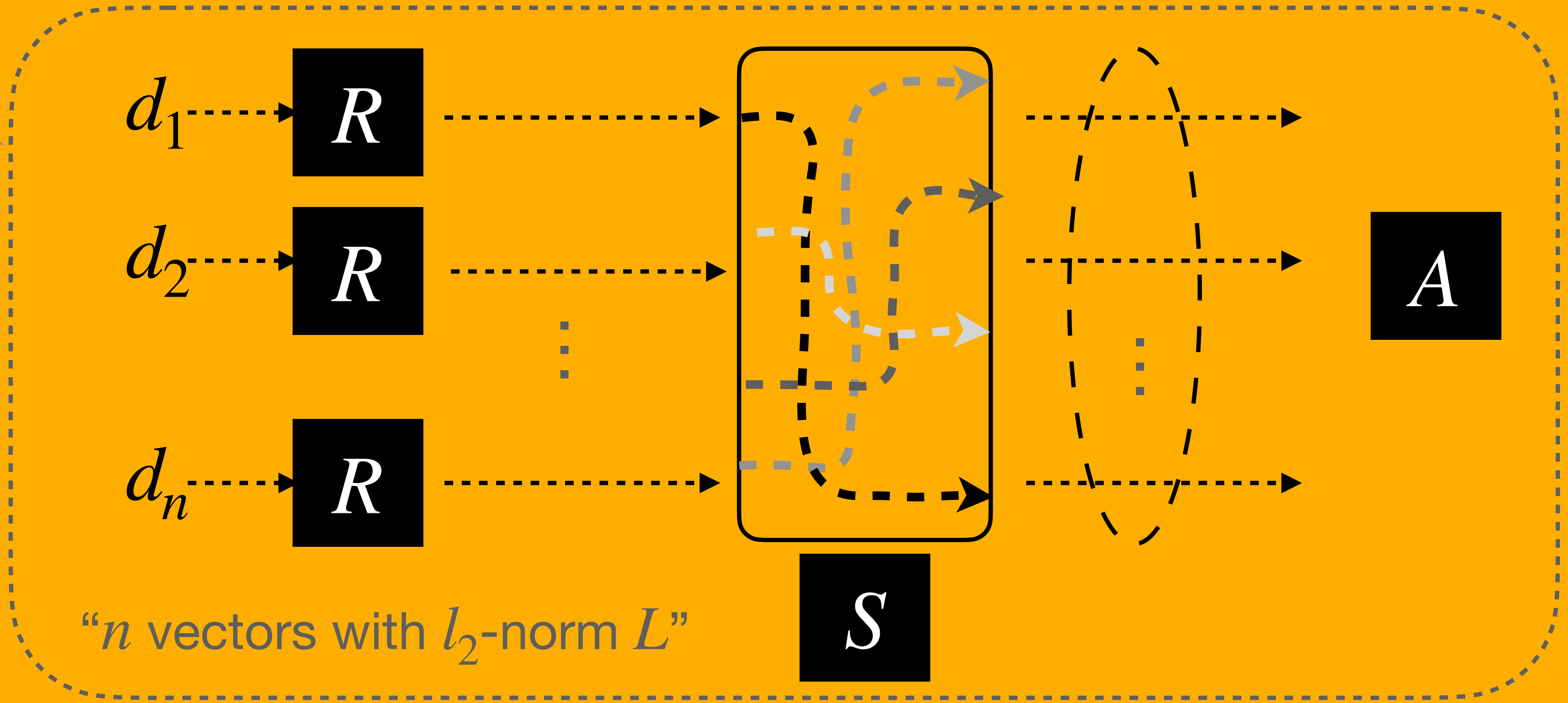
— each p-sum has $2B$ points

— $n = M \cdot 2B$ with each norm bounded

— each sync incurs only $1/\epsilon^2$ noise ✅

Differential Privacy 502

1. **How to achieve SDP?**
   — instead of using amplification lemma
   — one can use specific shuffle protocol
   — $P_{\text{vec}} = (R_{\text{vec}}, S, A_{\text{vec}})$[CJMP21] is one example



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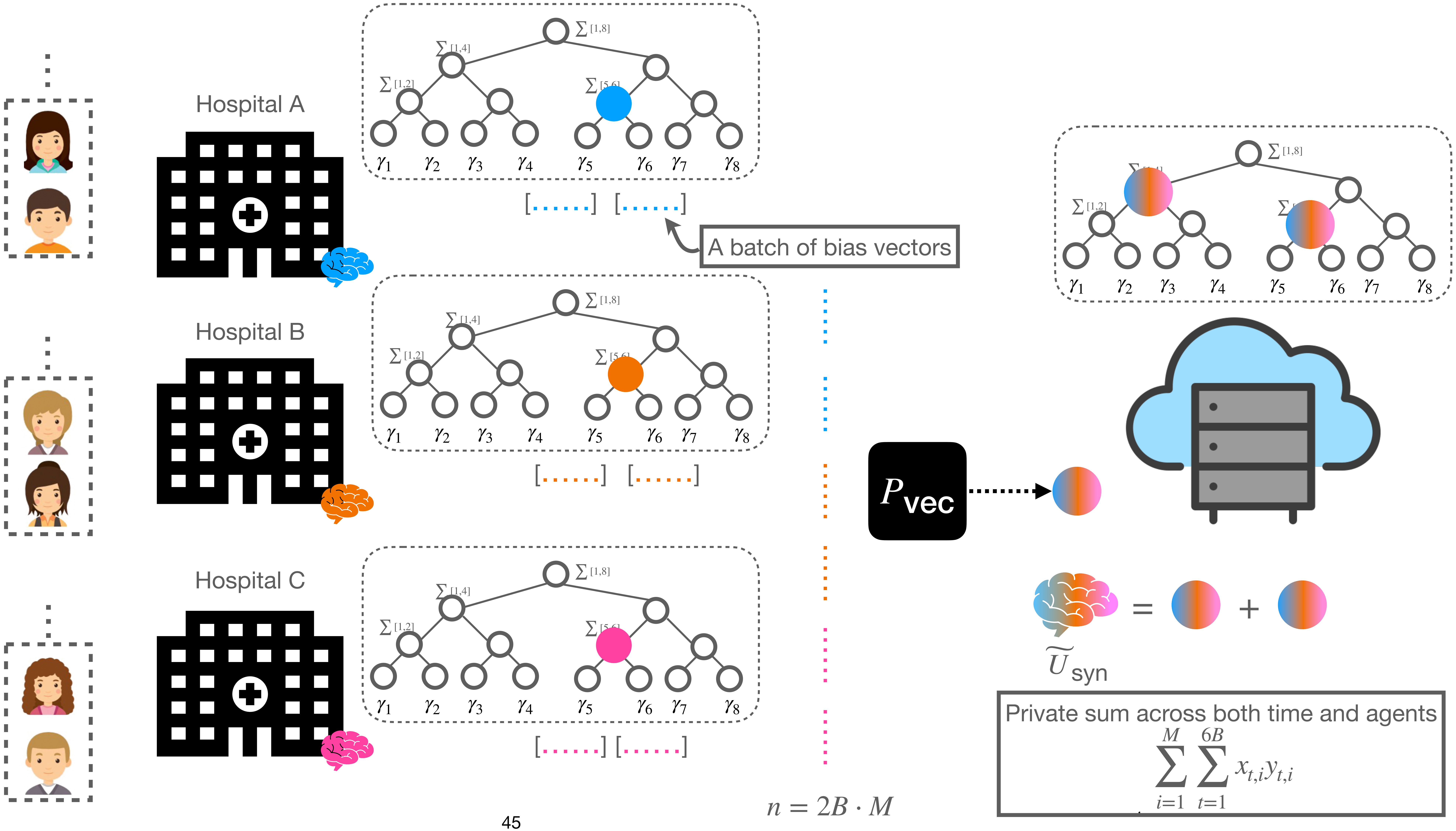
(Essentially, it simulates central model without a trusted server)

Algorithm in action

With P_{vec}

time $t = 6B$

(the 6-th communication)



Federated LCBs under SDP

Improved privacy via P_{vec}

Theorem 3 (Performance under SDP via P_{vec}, informal)

Let batch size $B = \sqrt{T/M}$ and $\kappa = 1 + \log(T/B)$. Combine P_{vec} with our privacy protocol. Then, `Private-FedLinUCB` enjoys

1. **Privacy** — (ϵ, δ)-SDP for any $\epsilon \in (0,60)$, $\delta \in (0,1)$,
2. **Regret** — $R_M(T) =$ non-private regret $+ \sqrt{MT}\dfrac{d^{3/4}\log^{3/4}(M\kappa/\delta)}{\sqrt{\epsilon}}$
3. **Communication** — $\sqrt{MT}$ rounds of sync between agents and server

Match the "lower bound"
Privacy cost is on the order of $\sqrt{MT}$

Privacy holds for a wide range of ϵ
This significantly improve upon the one via amplification

A more complicated algorithm
— need P_{vec}
— need store all data points

Analysis

A Generic Analysis

"One-line" proof for regret

Aggregated prefix sum
(sum over time and agents)

Privacy Noise Condition (PNC)

For any $t = kB$, let $N_{t,i}, n_{t,i}$ be total privacy noise injected in $\sum_{s=1}^{t} x_{s,i}x_{s,i}^\top$ and $\sum_{s=1}^{t} x_{s,i}y_{s,i}$, respectively

1. $\sum_{i \in [M]} n_{t,i}$ be a random vector, each entry is zero mean sub-Gaussian with variance at most σ_{tot}^2
2. $\sum_{i \in [M]} N_{t,i}$ be a random symmetric matrix, each entry is zero mean sub-Gaussian with variance at most σ_{tot}^2

Proposition 1 (Generic regret bound under PNC, informal)

Suppose that the privacy protocol satisfies PNC with parameter σ_{tot}^2, then `Private-FedLinUCB` enjoys the following regret with high probability

$$R_M(T) = \tilde{O}\left(dMB + d\sqrt{MT} + \sqrt{\sigma_{\text{tot}} MT} d^{3/4}\right)$$

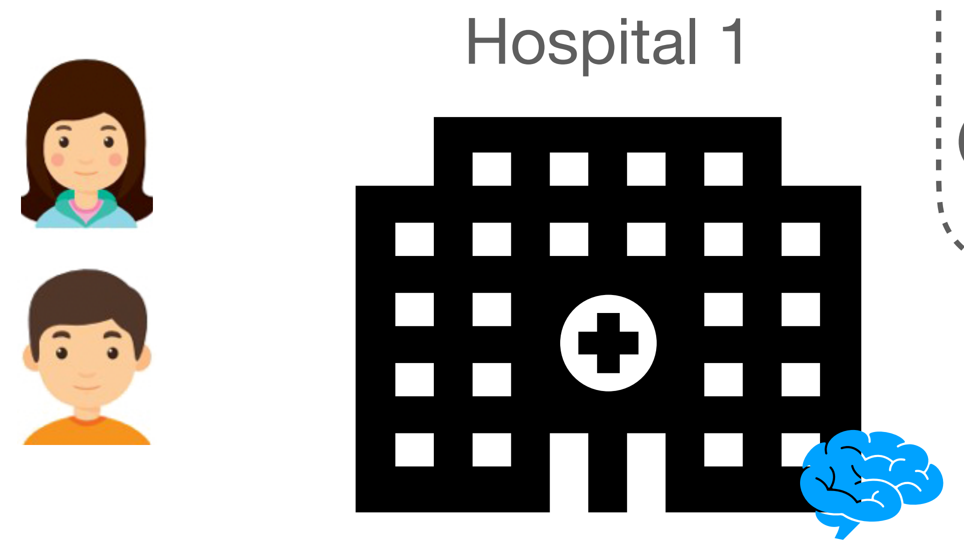
Cost due to batching | Standard regret | Cost due to privacy

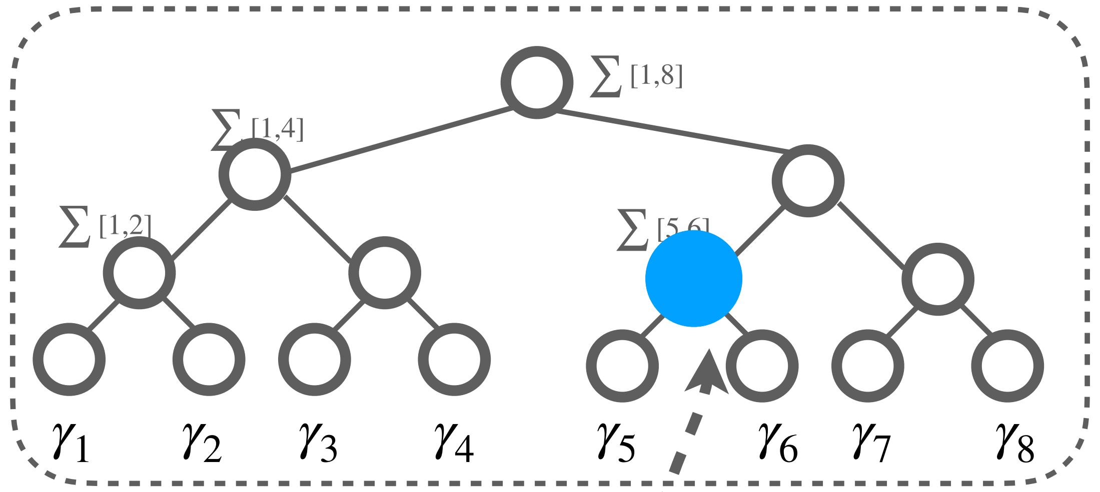
Total Privacy Noise

Silo-level LDP

Prop. 1. Regret due to privacy: $\sqrt{\sigma_{\text{tot}} M T}$

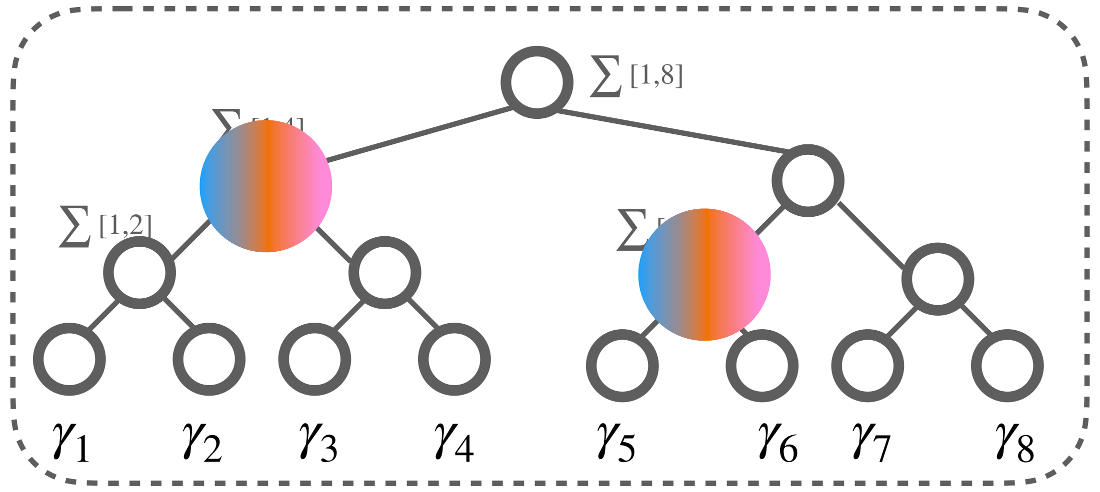
Regret under silo-level LDP: $\tilde{O}(M^{3/4}\sqrt{T/\epsilon})$





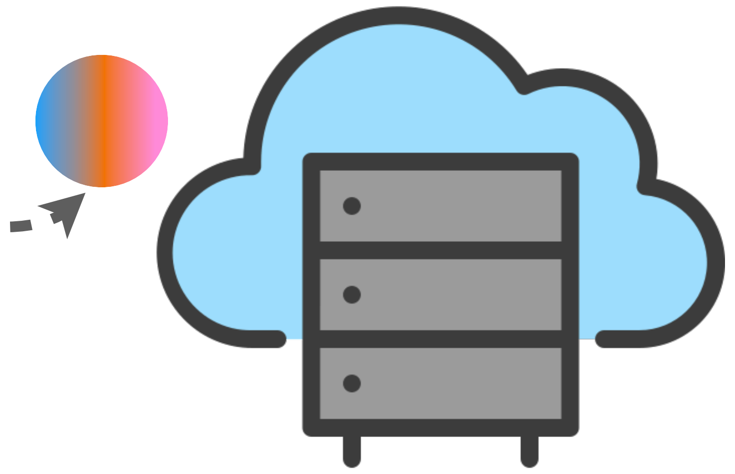
$\tilde{O}(1/\epsilon^2)$ privacy noise in **each** p-sum

(thanks to binary tree)



$\tilde{O}(M/\epsilon^2)$ privacy noise in **aggregated** p-sum

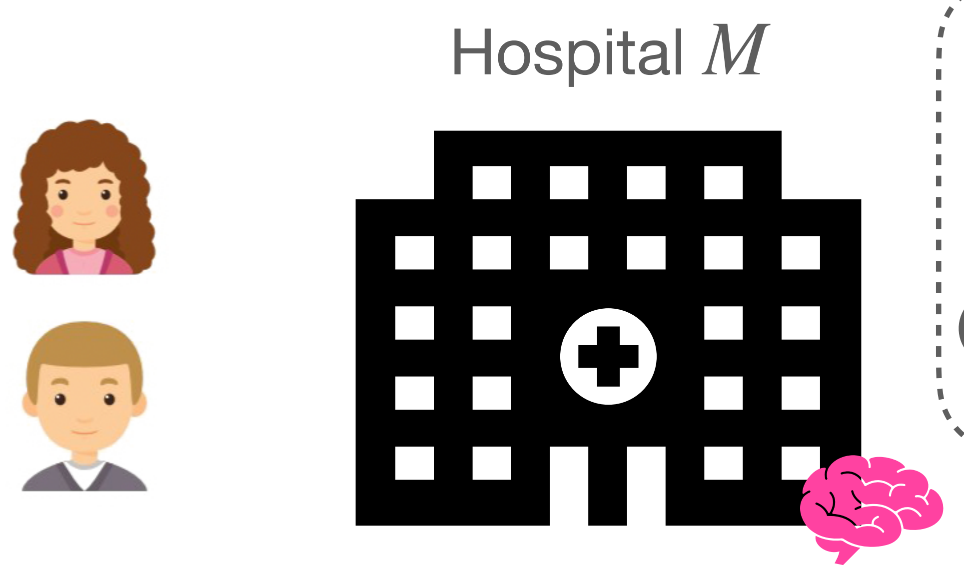
(sum of M noise)

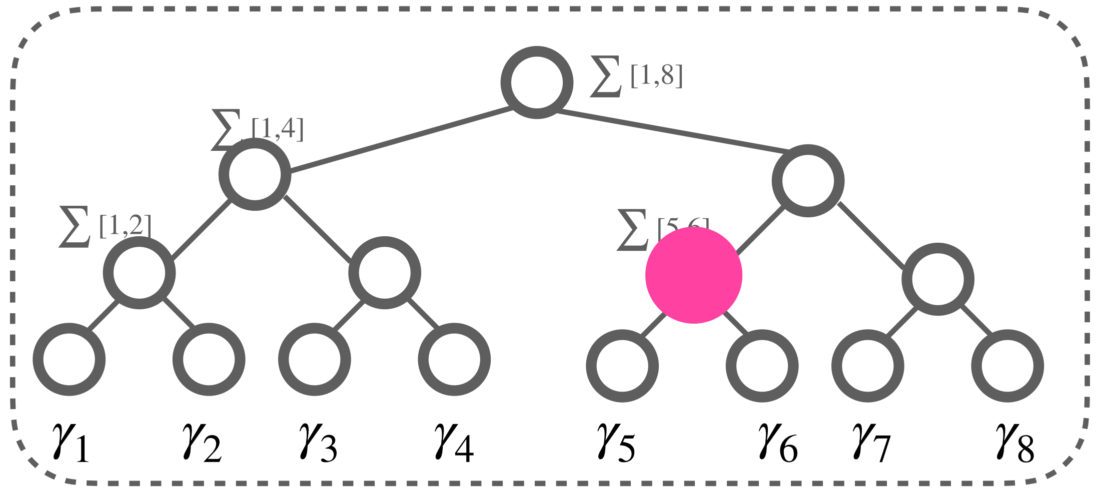


$\tilde{O}(M/\epsilon^2)$ privacy noise in **aggregated prefix** sum

i.e., σ_{tot}^2

(sum of $\log K$ noise)





Private sum across both time and agents

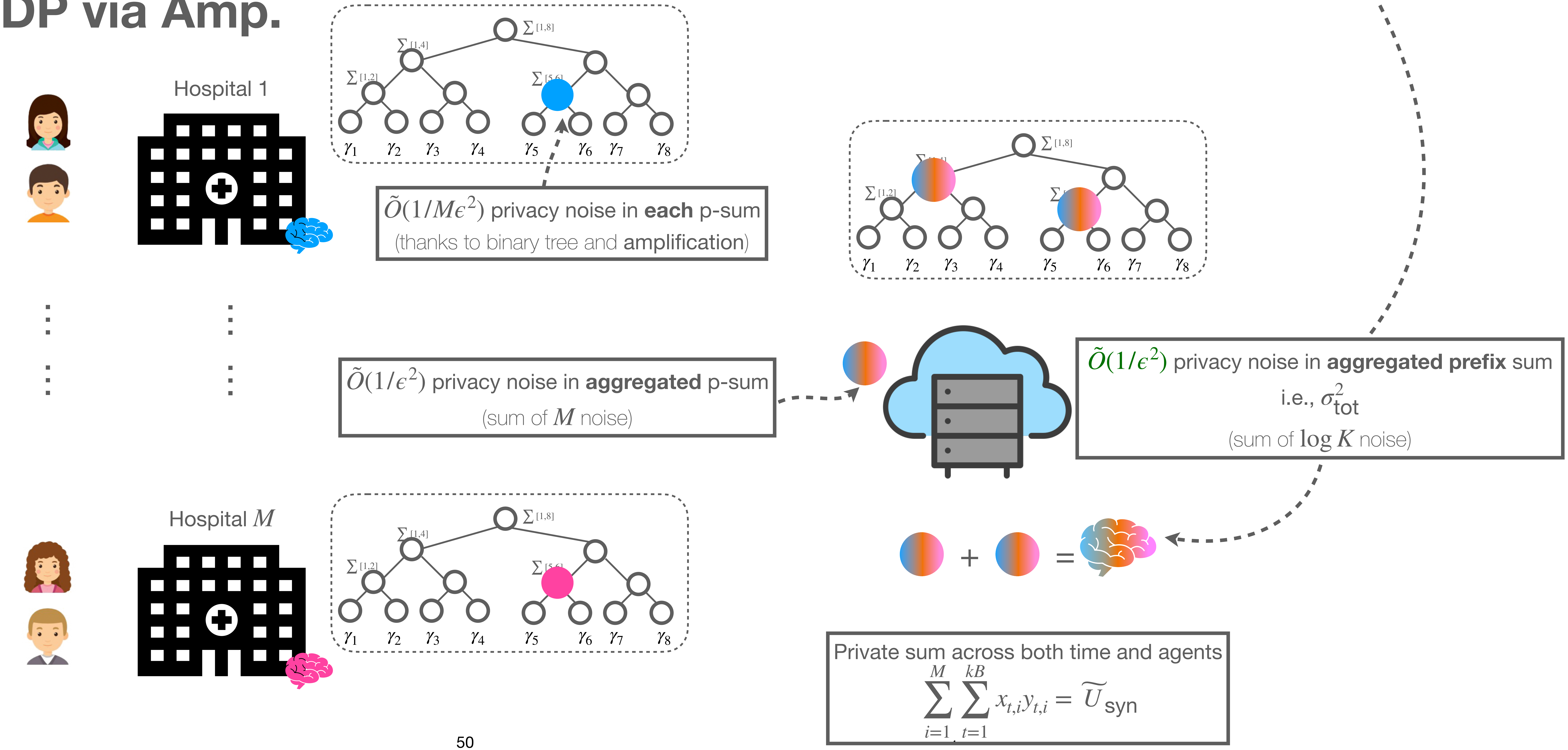
$$\sum_{i=1}^{M}\sum_{t=1}^{kB} x_{t,i} y_{t,i} = \widetilde{U}_{\text{syn}}$$

Total Privacy Noise

SDP via Amp.

Prop. 1. Regret due to privacy: $\sqrt{\sigma_{\text{tot}} MT}$

Regret under SDP: $\tilde{O}(\sqrt{MT/\epsilon})$

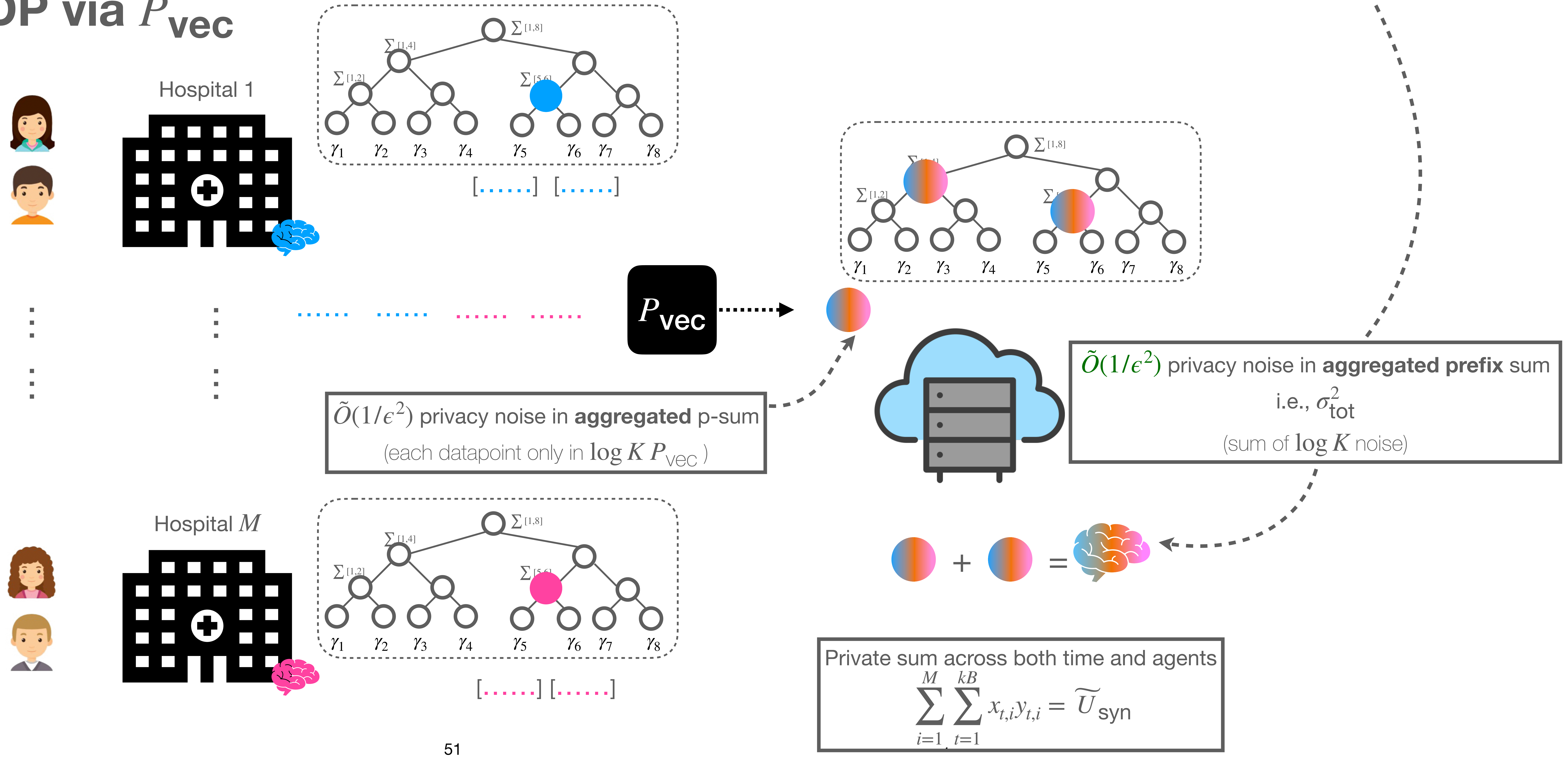


Total Privacy Noise

SDP via P_{vec}

Prop. 1. Regret due to privacy: $\sqrt{\sigma_{\text{tot}} MT}$

Regret under SDP: $\tilde{O}(\sqrt{MT/\epsilon})$



Importance of p-sum

Why P_{alt} fails for SDP

Prop. 1. Regret due to privacy: $\sqrt{\sigma_{\text{tot}} MT}$

Regret under SDP: $\tilde{O}(\sqrt{MT/\epsilon})$

Importance of p-sum

SDP via Amp.

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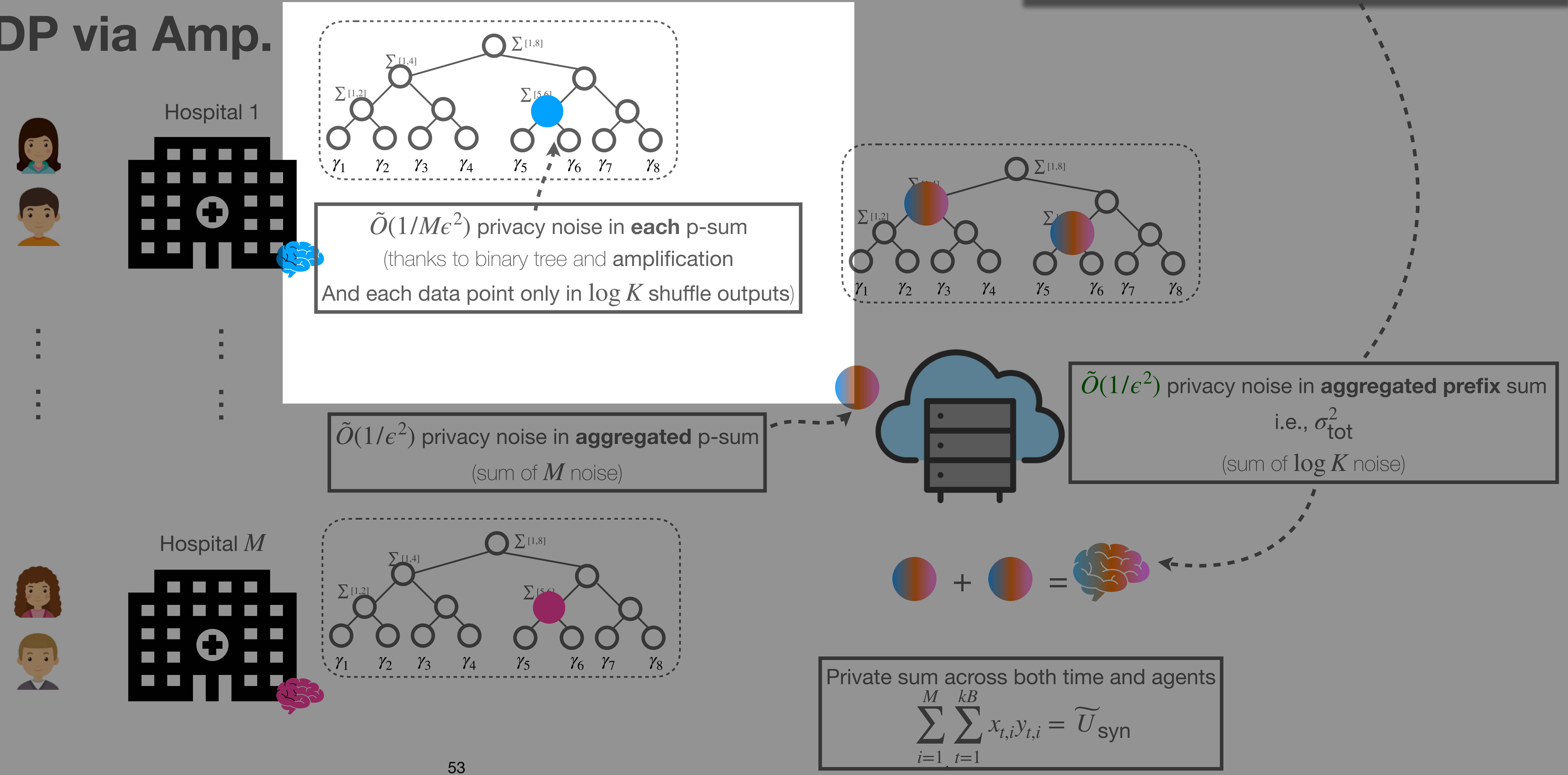

Importance of p-sum

SDP via Amp.

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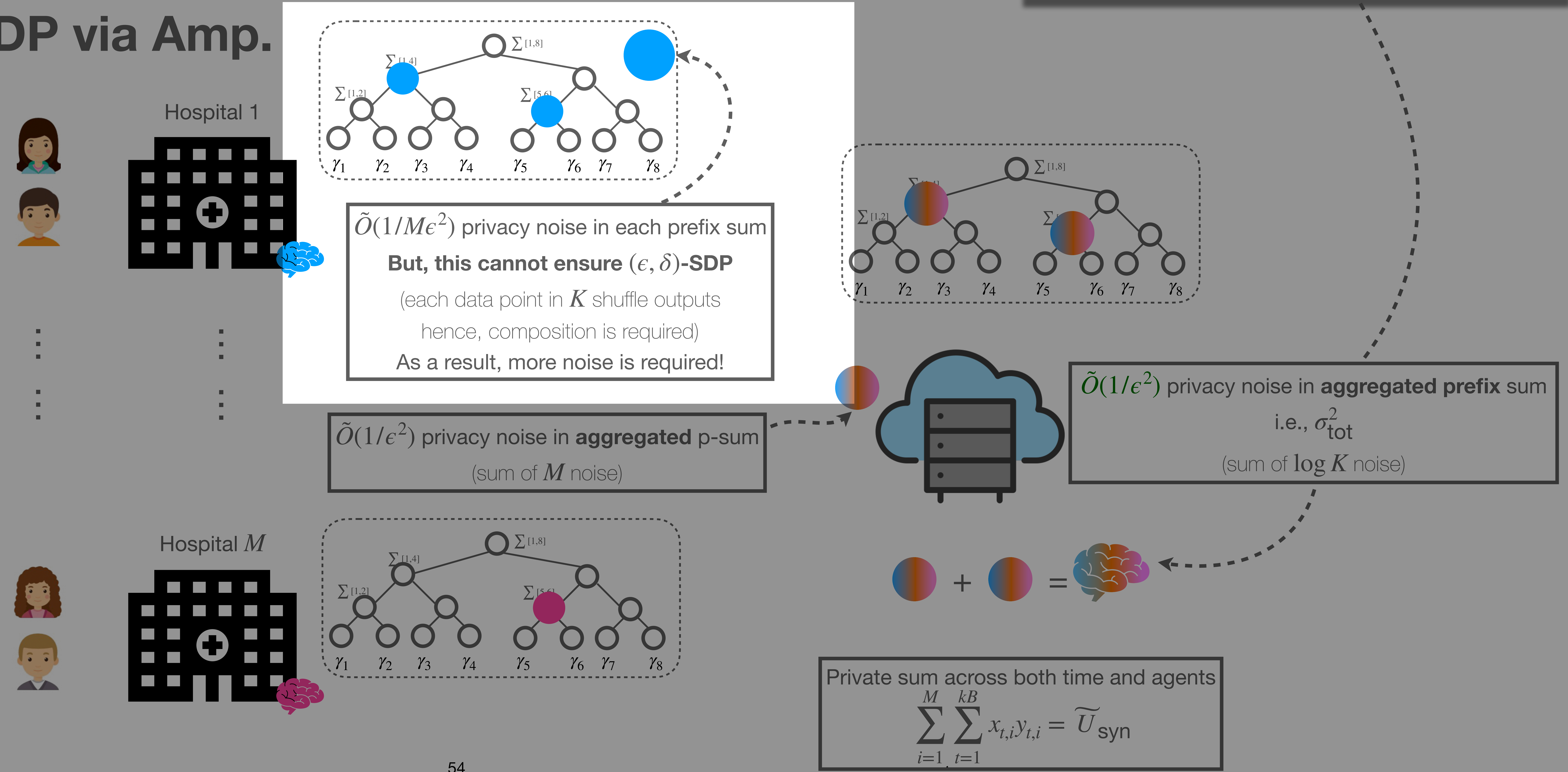
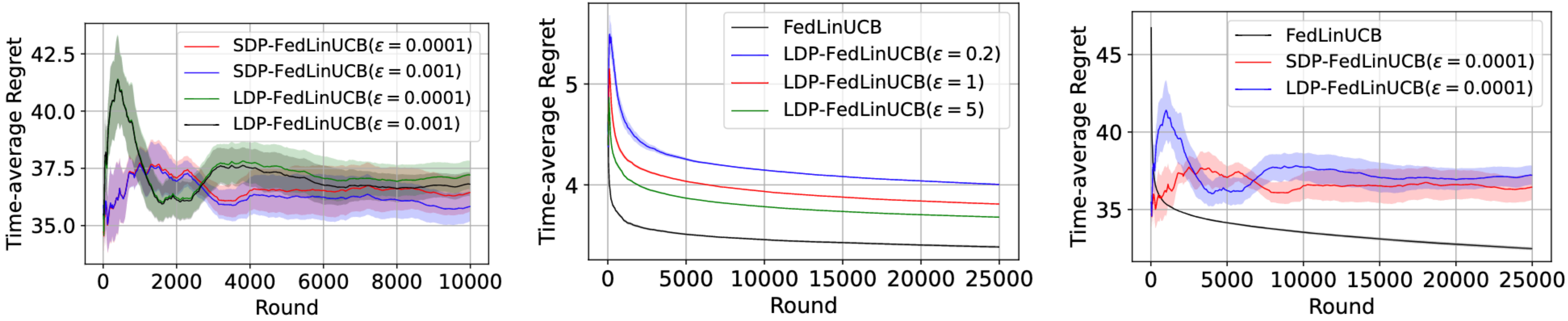


Simulations



(a) Synthetic data ($M = 100$) (b) Real data ($M = 10$) (c) Real data ($M = 100$)

Discussions

Q1: Can we further reduce comm. cost to $\log T$

Then, it might need adaptive update based on determinant condition. Challenges exist in private case

Q2: Silo-level LDP/ SDP vs. other privacy notions in contextual bandits?

We give a comprehensive discussions on difference and connections

Q3: What if users even do not trust each local agent?

It turns out that a simple tweak of our algorithm can handle this situation

Q4: What if users participate multiple times ? (within one silo or across silos)

One can use composition or group privacy to handle. Or directly analyze the total sensitivity

Q5: How to balance between privacy and algorithm complexity?

Good question. We are working on it right now

Q6: Can we generalize it to federated RL

Yes, at least for RL with linear function approximation

One last thing…

Recent Research…

Private Online Learning

- **Private MAB**
  - — “MAB under local DP with tight lower bound” [RZLS20, arxiv]
  - — “the state-of-the-art of private MAB for all three DP models” [CZ*23, ICLR23]
  - — “private and robust MAB” [WZ*TW23, submitted]
- **Private Contextual Bandits**
  - — “linear contextual bandits under shuffle model” [CZ*22, ICML22]
  - — “federated LCBs under both silo-level LDP and SDP [ZC, arixv, submitted]
  - — “kernel bandits under local model” [ZT21, AAAI21]
  - — “private linear bandits with distributed feedback” [LZJ22, WiOpt22, Best Student Paper]
  - — “private distributed kernel bandits” [LZJ23, Sigmetrics23]
- **Private RL**
  - — “A comprehensive study of tabular RL under both central and local DP models” [CZ*22, AAAI22, oral]
  - — “Private heavy-tailed RL”[WZ*CW23, ICML23]
  - — “The first study of private RL with linear function approximation”[Z22, Sigmetrics22]
  - — “Study of private LQR” [CZ*S21, ISIT21]

Many interesting open problems in this area…
Collaborations are welcome 🎉

[Z* means co-primary authors]

Recent Research…

Private Online Learning

- **Private MAB**
  - “MAB under local DP with tight lower bound” [RZLS20, arxiv]
  - “the state-of-the-art of private MAB for all three DP models” [CZ*23, ICLR23]
  - “private and robust MAB” [WZ*TW23, submitted]
- **Private Contextual Bandits**
  - “linear contextual bandits under shuffle model” [CZ*22, ICML22]
  - “federated LCBs under both silo-level LDP and SDP [ZC, arixv, submitted]
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Many interesting open problems in this area…
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Recent Research…

Bayesian Optimization (BO) and RL

How to make BO and RL more practical?

heavy-tailed rewards, non-stationarity, constraints

NeurIPS'22

On Kernelized Multi-Armed Bandits with Constraints

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AISTATS'22

Weighted Gaussian Process Bandits for Non-stationary Environments

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Xingyu Zhou
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Baekjin Kim
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University of Michigan

Abhishek Gupta
The Ohio State University

Ness Shroff
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NeurIPS'22

Provably Efficient Model-Free Constrained RL with Linear Function Approximation

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ICML'23

Differentially Private Episodic Reinforcement Learning with Heavy-tailed Rewards

Yulian Wu [1 2] **Xingyu Zhou** [3] **Sayak Ray Chowdhury** [4] **Di Wang** [1 2]

ICLR'23

ACHIEVING SUB-LINEAR REGRET IN INFINITE HORIZON AVERAGE REWARD CONSTRAINED MDP WITH LINEAR FUNCTION APPROXIMATION

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Sigmetrics'23

(Private) Kernelized Bandits with Distributed Biased Feedback

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Thank you!