Energy Efficient Relay Antenna Selection for AF MIMO Two-Way Relay Channels

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***Abstract*—In this paper, we investigate the energy efficiency (EE) maximization in amplify-and-forward (AF) MIMO two-way relay channel (TWRC) combined with relay antenna selection (AS). An iterative energy efficient AS algorithm is proposed to jointly select the active receive and transmit antennas at the relay, as well as optimize the transmission power of the sources and relay. Specifically, the AS at each iteration is based on a derived closed-form iterative equation of EE, which guides us to select a pair of receive and transmit relay antennas that achieves the largest increment of EE under an initial transmission power. After the AS of each iteration, a power adaptation is immediately adopted where we calculate the optimal transmission power using fractional programming and set it as the initial one for the AS of the next iteration. Simulation results show that our proposed scheme achieves nearly the same performance of exhaustive search while with significantly reduced complexity. Moreover, it is capable of simultaneously improving EE and reducing the transmission power.**

I. Introduction

Wireless multiple-input multiple-output (MIMO) relaying constitutes an appealing technique of improving the reliability, data rate and coverage for modern communication systems especially with correlated fading or shadowing [1], [2], [3]. Most recently, inter-relay interference cancellation and sub-carrier matching techniques have been exploited to further improve the spectral efficiency of cooperative relaying, respectively in [4] and [5]. In order to compensate the spectral efficiency loss in one-way relaying, AF MIMO relaying has been extended to two-way relay channel (TWRC) [6], [7]. However, the multiple RF chains associated with multiple antennas among all the nodes will consume more system energy when the circuit power consumption is considered [8]. Therefore, it becomes more and more important to design energy efficient transmission scheme for AF MIMO TWRCs.

Antenna selection (AS) in which only a subset of available antennas are active for transmitting and receiving maintains many advantages of MIMO relay systems while reducing the overall power consumption. Recently, AS has also been applied to two-way AF MIMO relaying [9]–[11]. In the previous works, the performance metric is either capacity or bit-error-ratio (BER) and only the transmission power is considered. However, from [8] we know that a holistic power model which considers the circuit power consumption as well as the transmission power should be adopted to comprehensively quantify the energy efficiency (EE) of wireless networks. In this case, the results of previous schemes cannot guarantee to be global energy efficient. Therefore, it's of vital importance for us to design AF MIMO relaying from the perspective of EE, which is typically defined as the ratio of the achievable rate to the overall power consumption. The authors in [12] investigated the EE maximization in one-way AF MIMO relay systems with both perfect and statistical channel state information (CSI) without the consideration of AS. The problem of EE optimization combined with relay AS in one-way AF MIMO relay systems was studied in our previous work [13].

The motivation for this paper is twofold. Firstly, although there have been AS algorithms for AF MIMO TWRCs, few works consider the circuit power consumption and hence the number of active RF chains is fixed rather than dynamical. Secondly, the EE maximization in AF MIMO TWRCs has not been investigated in a systematical way. Thereby, in this paper, we investigate the EE maximization combined with relay AS in AF MIMO TWRCs when the circuit power consumption is considered. Specifically, our task is to find the optimal number of active RF chains, the corresponding receive and transmit relay antenna subsets as well as the optimal transmission power of the two sources and relay. Since exhaustive search for this NP-hard (non-deterministic polynomial-time hard) problem is complexity prohibitive, an iterative EE maximization relay AS algorithm relying on a joint iteration of active antennas and transmission power is proposed in this paper. In particular, we first derive a closed-form iterative equation for the EE in AF MIMO TWRCs with relay AS under the holistic power model. Based on this iterative property, we select one pair of receive and transmit antennas at the relay that achieves the largest increment of EE for an initial transmission power tuple at each iteration. Following the update of selected antennas at each iteration, we update the transmission power by using the tool of fractional programming and set the new transmission power tuple as the initial one for the AS of the next iteration. It is worth

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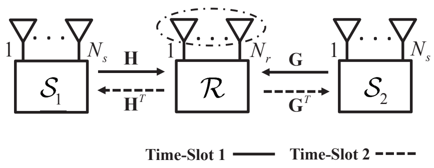

Fig. 1. System Model of AF MIMO TWRC with Relay AS.

pointing out that the power adaptation method itself is another contribution, which could be adopted to optimize the EE for AF MIMO TWRCs without AS under given rate requirement. Simulation results show that our proposed scheme achieves nearly the same performance of exhaustive search while with significant reduced computational complexity. Moreover, since our scheme focuses on energy efficient AS and considers the dynamical change in the number of active RF chains, it outperforms conventional AS schemes in terms of both the EE and transmission power. In addition, simulation results verify that the proposed power adaptation method alone is capable of improving the EE for AF MIMO TWRCs without AS.

The rest of the paper is structured as follows. The system model and the problem formulation are given in Section II. In section III, the closed-form iterative equation for the EE and the power adaptation method are respectively investigated. Then we propose the energy efficient AS and analyze its computational complexity. Finally, simulation results and conclusions are presented in Sections IV and V, respectively.

II. System Model and Problem Formulation

A. System Model

As shown in Fig. 1, we consider an AF MIMO TWRC system consisting of two sources with N_s antennas each, $\mathcal{S}_1$ and $\mathcal{S}_2$, and a relay $\mathcal{R}$ with N_r antennas. The direct link between $\mathcal{S}_1$ and $\mathcal{S}_2$ is ignored due to large path loss effect. The channel matrix from $\mathcal{S}_1$ to $\mathcal{R}$ is denoted by $\mathbf{H}$, and the channel matrix from $\mathcal{S}_2$ to $\mathcal{R}$ is denoted by $\mathbf{G}$. Channel reciprocity is herein assumed to hold for TWRC; thus, the channels from $\mathcal{R}$ to $\mathcal{S}_1$ and $\mathcal{S}_2$ are given by $\mathbf{H}^T$ and $\mathbf{G}^T$, respectively. Assume that all the channels are Rayleigh flat fading with average power varied with the path loss. Throughout the paper, $\mathcal{S}_1$ and $\mathcal{S}_2$ always use all N_s antennas. In contrast, the number of active antennas or RF chains at $\mathcal{R}$ is adaptive and is determined by the results of AS. In particular, the selected subsets of active receive and transmit antennas at $\mathcal{R}$ are respectively denoted by ω_r and ω_t. $L = |\omega_r| = |\omega_t|$[1] is the number of active antennas used for reception and transmission at $\mathcal{R}$. It's worth pointing that L is not given and fixed in advance; rather, it can range from 1 to N_r and is a key parameter that should be carefully optimized in this paper. The remaining receive and transmit antenna subsets are respectively given by $\omega_r^c = \mathcal{N} - \omega_r$ and $\omega_t^c = \mathcal{N} - \omega_t$, where $\mathcal{N} = \{1, 2, \ldots, N_r\}$. Furthermore, we assume that $\mathcal{R}$ operates in half-duplex mode, and thus ω_r and ω_t are independent with each other, i.e., the selected active antenna subsets for reception and transmission are not necessarily the same. The transmission protocol uses two time-slots for an exchange messages between $\mathcal{S}_1$ and $\mathcal{S}_2$.

In the first time-slot of AF TWRC, $\mathcal{S}_1$ and $\mathcal{S}_2$ simultaneously transmit their own signals to $\mathcal{R}$, which receives the signal from the antennas in ω_r. During the second time-slot, $\mathcal{R}$ processes the received signal with AF relay operation[2], and then broadcasts the processed signal to $\mathcal{S}_1$ and $\mathcal{S}_2$ by using the antennas in ω_t. With the adoption of analogue network coding (ANC) and perfect channel state information (CSI), the achievable rate from $\mathcal{S}_1$ to $\mathcal{S}_2$, and the achievable rate from $\mathcal{S}_2$ to $\mathcal{S}_1$ are respectively given by

$$R_1 = \frac{1}{2}\log_2 \frac{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_{\omega_t}^T (\mathbf{G}_{\omega_t}^T)^H + \alpha\rho_1 \mathbf{F}_1 \mathbf{F}_1^H\right|}{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_{\omega_t}^T (\mathbf{G}_{\omega_t}^T)^H\right|}, \quad (1)$$

$$R_2 = \frac{1}{2}\log_2 \frac{\left|\mathbf{I}_{N_s} + \alpha \mathbf{H}_{\omega_t}^T (\mathbf{H}_{\omega_t}^T)^H + \alpha\rho_2 \mathbf{F}_2 \mathbf{F}_2^H\right|}{\left|\mathbf{I}_{N_s} + \alpha \mathbf{H}_{\omega_t}^T (\mathbf{H}_{\omega_t}^T)^H\right|}, \quad (2)$$

where $\mathbf{F}_1 = \mathbf{G}_{\omega_t}^T \mathbf{H}_{\omega_r}$ and $\mathbf{F}_2 = \mathbf{H}_{\omega_t}^T \mathbf{G}_{\omega_r}$. The subscripts ω_r and ω_t here are used to represent the corresponding subchannels; that is, $\mathbf{H}_{\omega_r}$ and $\mathbf{G}_{\omega_r}$ are the $|\omega_r| \times N_s$ subchannel matrices from $\mathcal{S}_1$ to $\mathcal{R}$ and from $\mathcal{S}_2$ to $\mathcal{R}$ respectively when $\mathcal{R}$ activates the antennas in ω_r for reception. $\mathbf{H}_{\omega_t}^T$ and $\mathbf{G}_{\omega_t}^T$ respectively represent the $N_s \times |\omega_t|$ subchannel matrices from $\mathcal{R}$ to $\mathcal{S}_1$ and $\mathcal{S}_2$ when $\mathcal{R}$ activates the antennas in ω_t for transmission. $\alpha = \frac{P_r/N_0}{\rho_1 \mathrm{tr}\left(\mathbf{H}_{\omega_r}\mathbf{H}_{\omega_r}^H\right) + \rho_2 \mathrm{tr}\left(\mathbf{G}_{\omega_r}\mathbf{G}_{\omega_r}^H\right) + |\omega_r|}$ is the power coefficient satisfying the relay power constraint P_r, where $\rho_1 = P_{s,1}/(N_s N_0)$ and $\rho_2 = P_{s,2}/(N_s N_0)$. $P_{s,1}$ and $P_{s,2}$ are the transmission power at $\mathcal{S}_1$ and $\mathcal{S}_2$, and N_0 is the additive noise power. Therefore, the achievable sum rate of AF MIMO TWRCs with relay AS is given by

$$R_{\text{sum}} = \omega_1 R_1 + \omega_2 R_2, \quad (3)$$

where $\omega_1 = \omega_2 = 1$ is our considered case in the paper.

The overall power consumption in the considered system consists of two main parts: the power consumed by the amplifiers and the power consumed by all the RF circuits. The second part is given by [14], [15],

$$P_c = 2N_s\left(P_{ct,S} + P_{cr,S}\right) + |\omega_r|\left(P_{cr,R} + P_{ct,R}\right) + P_{c0}, \quad (4)$$

where $P_{ct,S}$ and $P_{ct,R}$ denote the power consumed by each source and relay RF chain for transmission. $P_{cr,S}$ and $P_{cr,R}$ are the power consumption of each source and relay RF chain for reception. P_{c0} represents the power consumed by frequency synthesizers and other circuit units. Therefore, the overall power consumption in the system can be expressed as

$$P = \frac{1}{\eta_s}(P_{s,1} + P_{s,2}) + \frac{1}{\eta_r} P_r + P_c, \quad (5)$$

where η_s and η_r are the drain efficiency of the power amplifier at the source and relay, respectively.

[1]This equation constraint holds because each RF chain at the relay actually connects one receive antenna and one transmit antenna, as shown in [14]. Therefore, once we activate one more RF chain at the relay, we actually activate a pair of receive and transmit antennas.

[2]We do not consider the AS under the optimal relay matrix here since the optimal relay matrix obtained by the iterative methods would make the AS process intractable. Therefore, we only focus on the AS process under a scaled identity matrix, which is also adopted in [9].

B. Problem Formulation

The main objective of the paper is to maximize the EE for AF MIMO TWRC with AS adopted at $\mathcal{R}$. According to the definiton of EE in [8], the EE of the AF MIMO TWRC in this paper is given by

$$EE = \frac{R_{\text{sum}}}{\frac{1}{\eta_s}(P_{s,1}+P_{s,2}) + \frac{1}{\eta_r}P_r + 2N_sP_{c,S} + |\omega_r|\,P_{c,R} + P_{c0}}, \tag{6}$$

where $P_{c,S} = P_{ct,S}+P_{cr,S}$, and $P_{c,R} = P_{ct,R}+P_{cr,R}$. In order to maximize the EE defined in Eq. (6), a joint optimization over the transmission power and the active antennas subsets at $\mathcal{R}$ is needed. Mathematically, our objective can be equivalently represented by

$$\max_{(P_{s,1},P_{s,2},P_r,\omega_r,\omega_t)} EE$$
$$s.t. \begin{cases} R_{\text{sum}} \geq R_{\min} \\ 1 \leq |\omega_r| = |\omega_t| \leq N_r \\ 0 < P_r \leq P_r^{\max}, 0 < P_{s,k} \leq P_s^{\max}, k \in \{1,2\}. \end{cases} \tag{7}$$

In problem (7), $R_{\min}$ is the minimum required rate. $P_s^{\max}$ and $P_r^{\max}$ are the maximum transmission power for the source and relay, respectively.

III. Optimization of the Energy Efficiency with AS

In this section, an iterative energy efficient relay AS algorithm is proposed to solve the problem defined in Eq. (7). The main idea of this algorithm has two key points: 1) At each iteration, it judiciously selects a pair of receive and transmit relay antennas that achieves the largest increment of EE for a given transmission power tuple. 2) After the AS of each iteration, it calculates the optimal transmission power tuple of the sources and relay for the current selected antenna subsets and set it as the initial transmission power tuple for the next iteration. The two key points are respectively elaborated in subsections III-A and III-B. Furthermore, complexity analysis of different algorithms is presented in subsection III-C.

A. Antenna Selection under Given Transmission Power

The AS process under any given transmission power relies on Theorem 1, in which we give the closed-form iterative equation of EE in AF TWRC when a pair of relay receive and transmit antenna is selected at each iteration. For clarity, let us first introduce the notations used in Theorem 1.

The selected active receive and transmit antenna subsets after n iterations are respectively denoted by $\omega_r^{(n)} = \{r(1), r(2), \ldots, r(n)\}$ and $\omega_t^{(n)} = \{t(1), t(2), \ldots, t(n)\}$, where $r(k)$ and $t(k)$ stand for the indices for the kth selected receive and the kth selected transmit antenna, respectively. Meanwhile, the subchannel matrices from $\mathcal{S}_1$ to $\mathcal{R}$ and from $\mathcal{S}_2$ to $\mathcal{R}$ after n iterations are respectively denoted by $\mathbf{H}_{\omega_r^{(n)}}$ and $\mathbf{G}_{\omega_r^{(n)}}$; subchannel matrices from $\mathcal{R}$ to $\mathcal{S}_1$ and $\mathcal{S}_2$ are respectively denoted by $\mathbf{H}^T_{\omega_t^{(n)}}$ and $\mathbf{G}^T_{\omega_t^{(n)}}$. For convenience, let $\mathbf{\Psi}_1^{(n)} = |\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{G}}^{(n)}|$ and $\mathbf{\Gamma}_1^{(n)} = |\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n)}\widetilde{\mathbf{F}}_1^{(n)}|$ represent the denominator and numerator of Eq. (1), and $\mathbf{\Psi}_2^{(n)} = |\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{H}}^{(n)}|$ and $\mathbf{\Gamma}_2^{(n)} = |\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{H}}^{(n)} + \beta_2^{(n)}\widetilde{\mathbf{F}}_2^{(n)}|$ represent the denominator and numerator of Eq. (2) after n iterations, where $\widetilde{\mathbf{G}}^{(n)} = \mathbf{G}^T_{\omega_t^{(n)}}(\mathbf{G}^T_{\omega_t^{(n)}})^H$, $\widetilde{\mathbf{H}}^{(n)} = \mathbf{H}^T_{\omega_t^{(n)}}(\mathbf{H}^T_{\omega_t^{(n)}})^H$, $\widetilde{\mathbf{F}}_1^{(n)} = \mathbf{F}_1^{(n)}(\mathbf{F}_1^{(n)})^H$, $\mathbf{F}_1^{(n)} = \mathbf{G}^T_{\omega_t^{(n)}}\mathbf{H}_{\omega_r^{(n)}}$, $\widetilde{\mathbf{F}}_2^{(n)} = \mathbf{F}_2^{(n)}(\mathbf{F}_2^{(n)})^H$, $\mathbf{F}_2^{(n)} = \mathbf{H}^T_{\omega_t^{(n)}}\mathbf{G}_{\omega_r^{(n)}}$, $\beta_1^{(n)} = \alpha^{(n)}\rho_1$, $\beta_2^{(n)} = \alpha^{(n)}\rho_2$ and $\alpha^{(n)} = \frac{P_r/N_0}{\rho_1 \mathrm{tr}\left(\mathbf{H}_{\omega_r^{(n)}}\mathbf{H}^H_{\omega_r^{(n)}}\right) + \rho_2\mathrm{tr}\left(\mathbf{G}_{\omega_r^{(n)}}\mathbf{G}^H_{\omega_r^{(n)}}\right) + n}$. The overall power consumption for the nth iteration is denoted by $P^{(n)}$, i.e., the number of active RF chains is equal to n. According to Eq. (5), we have $P^{(n)} = nP_{c,R} + \frac{1}{\eta_s}(P_{s,1}+P_{s,2}) + \frac{1}{\eta_r}P_r + 2N_sP_{c,S} + P_{c0}$. At the $(n+1)$th iteration, if the l^*th receive and the s^*th transmit antenna at the relay are selected, that is, $r(n+1) = l^*$, $t(n+1) = s^*$, the new channel matrices are denoted by $\mathbf{H}_{\omega_r^{(n+1)}}$, $\mathbf{G}_{\omega_r^{(n+1)}}$, $\mathbf{H}^T_{\omega_t^{(n+1)}}$ and $\mathbf{G}^T_{\omega_t^{(n+1)}}$. $\mathbf{h}_l$ is the transpose of the lth row of $\mathbf{H}$ and $\mathbf{g}_s$ is the sth column of $\mathbf{G}^T$.

Theorem 1: With the relay antenna selection and given transmission power, the EE of AF MIMO TWRC under the holistic power model is given by the following iterative equation[3]

$$EE^{(n+1)} = \Lambda^{(n)}EE^{(n)} + D^{(n+1)} + \Delta_{(l,s)}^{(n+1)}, \tag{8}$$

where

$$\Lambda(n) = \frac{P^{(n)}}{P^{(n+1)}}, \tag{9}$$

$$D^{(n+1)} = \frac{1}{2P^{(n+1)}}\log_2 \frac{\zeta_{\gamma_1,2}^{(n+1)}\zeta_{\gamma_1,1}^{(n+1)}\zeta_{\gamma_2,2}^{(n+1)}\zeta_{\gamma_2,1}^{(n+1)}}{\zeta_{\psi_1,1}^{(n+1)}\zeta_{\psi_2,1}^{(n+1)}}, \tag{10}$$

$$\Delta_{(l,s)}^{(n+1)} = \frac{1}{2P^{(n+1)}}\log_2 \frac{\delta_{\gamma_1,(l,s)}^{(n+1)}\delta_{\gamma_2,(l,s)}^{(n+1)}}{\delta_{\psi_1,s}^{(n+1)}\delta_{\psi_2,s}^{(n+1)}}. \tag{11}$$

Proof: According to Eq. (6), the EE at the $(n+1)$th iteration can be expressed by

$$EE^{(n+1)} = \frac{R_1^{(n+1)} + R_2^{(n+1)}}{P^{(n+1)}}, \tag{12}$$

where $R_1^{(n+1)} = \frac{1}{2}\log_2\frac{\mathbf{\Gamma}_1^{(n+1)}}{\mathbf{\Psi}_1^{(n+1)}}$ and $R_2^{(n+1)} = \frac{1}{2}\log_2\frac{\mathbf{\Gamma}_2^{(n+1)}}{\mathbf{\Psi}_2^{(n+1)}}$. Thus, in order to obtain the iterative equation for the EE, we have to first find the iterative equations for $\mathbf{\Psi}_k^{(n+1)}$ and $\mathbf{\Gamma}_k^{(n+1)}$, $k \in \{1,2\}$. Here we take the derivation of $\mathbf{\Psi}_1^{(n+1)}$ and $\mathbf{\Gamma}_1^{(n+1)}$ as an example, and the same process can be applied to the other two.

First, we aim to find the iterative equation for $\mathbf{\Psi}_1^{(n+1)} = \left|\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n+1)}\right|$. Consider the selection of transmit antenna at $\mathcal{R}$ in the second time-slot, the channel matrix update is given by

$$\widetilde{\mathbf{G}}^{(n+1)} = \widetilde{\mathbf{G}}^{(n)} + \mathbf{g}_s\mathbf{g}_s^H. \tag{13}$$

[3]Due to space limitation, $\delta_{\psi_1,s}^{(n+1)}$, $\zeta_{\psi_1,1}^{(n+1)}$, $\delta_{\gamma_1,(l,s)}^{(n+1)}$, $\zeta_{\gamma_1,1}^{(n+1)}$, $\zeta_{\gamma_1,2}^{(n+1)}$ are respectively defined in Eqs. (15), (19). (22), (25) and (26). $\delta_{\psi_2,s}^{(n+1)}$, $\zeta_{\psi_2,1}^{(n+1)}$, $\delta_{\gamma_2,(l,s)}^{(n+1)}$, $\zeta_{\gamma_2,1}^{(n+1)}$, $\zeta_{\gamma_2,2}^{(n+1)}$ can be obtained in the same way, and hence the definitions are omitted.

By substituting Eq. (13) into $\mathbf{\Psi}_1^{(n+1)}$ and applying the matrix determinant lemma, we can arrive at

$$\mathbf{\Psi}_1^{(n+1)} = \left|\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)}\right| \delta_{\psi_1,s}^{(n+1)}, \tag{14}$$

where

$$\delta_{\psi_1,s}^{(n+1)} = 1 + \alpha^{(n+1)}\mathbf{g}_s^H \mathbf{T}_{\psi_1}^{(n)} \mathbf{g}_s, \tag{15}$$

$$\mathbf{T}_{\psi_1}^{(n)} = \left(\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)}\right)^{-1}. \tag{16}$$

In order to extract the term $\mathbf{\Psi}_1^{(n)}$ from Eq. (14), we can first rewrite Eq. (14) as follows

$$\mathbf{\Psi}_1^{(n+1)} = \left|\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{G}}^{(n)} - \tau^{(n)}\widetilde{\mathbf{G}}^{(n)}\right| \delta_{\psi_1,s}^{(n+1)}, \tag{17}$$

where $\tau^{(n)} = \alpha^{(n)} - \alpha^{(n+1)}$. Now $\mathbf{\Psi}_1^{(n)}$ can be easily extracted from Eq. (17) by using the generalization of matrix determinant lemma.

$$\mathbf{\Psi}_1^{(n+1)} = \mathbf{\Psi}_1^{(n)} \zeta_{\psi_1,1}^{(n+1)} \delta_{\psi_1,s}^{(n+1)}, \tag{18}$$

where

$$\zeta_{\psi_1,1}^{(n+1)} = \left|\mathbf{I} - \tau^{(n)}\mathbf{G}_{\omega_t^{(n)}}^{*} \left(\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{G}}^{(n)}\right)^{-1} \mathbf{G}_{\omega_t^{(n)}}^{T}\right|. \tag{19}$$

Having obtained the iterative equation for $\mathbf{\Psi}_1^{(n+1)}$, we can now proceed analogously to find the iterative equation for $\mathbf{\Gamma}_1^{(n+1)} = \left|\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n+1)} + \beta_1^{(n+1)}\widetilde{\mathbf{F}}_1^{(n+1)}\right|$.

Substituting the matrix updates of $\widetilde{\mathbf{G}}^{(n+1)}$ and $\widetilde{\mathbf{F}}_1^{(n+1)}$ into $\mathbf{\Gamma}_1^{(n+1)}$, we can obtain

$$\mathbf{\Gamma}_1^{n+1} = \left|\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n+1)}\widetilde{\mathbf{F}}_1^{(n)} + \mathbf{g}_s\mathbf{q}^H + \mathbf{q}\mathbf{g}_s^H\right|, \tag{20}$$

where $\mathbf{q} = \frac{\alpha^{(n+1)} + \beta_1^{(n+1)}\|\mathbf{h}_l\|^2}{2}\mathbf{g}_s + \beta_1^{(n+1)}\mathbf{F}_1^{(n)}\mathbf{h}_l^*$. We can simplify Eq. (20) with the rank-2 update property of determinant:

$$\mathbf{\Gamma}_1^{(n+1)} = \left|\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n+1)}\widetilde{\mathbf{F}}_1^{(n)}\right| \delta_{\gamma_1,(l,s)}^{(n+1)}, \tag{21}$$

where

$$\delta_{\gamma_1,(l,s)}^{(n+1)} = \left(1 + \mathbf{q}^H\mathbf{T}_{\gamma_1}^{(n)}\mathbf{g}_s\right)^2 - \left(\mathbf{g}_s^H\mathbf{T}_{\gamma_1}^{(n)}\mathbf{g}_s\right)\left(\mathbf{q}^H\mathbf{T}_{\gamma_1}^{(n)}\mathbf{q}\right), \tag{22}$$

$$\mathbf{T}_{\gamma_1}^{(n)} = \left(\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n+1)}\widetilde{\mathbf{F}}_1^{(n)}\right)^{-1}. \tag{23}$$

To extract the term $\mathbf{\Gamma}_1^{(n)}$ from Eq. (21), we apply the updates of $\alpha^{(n+1)}$ and $\beta_1^{(n+1)}$ and the generalization of matrix determinant lemma to Eq. (21) and hence have

$$\mathbf{\Gamma}_1^{(n+1)} = \mathbf{\Gamma}_1^{(n)} \zeta_{\gamma_1,2}^{(n+1)} \zeta_{\gamma_1,1}^{(n+1)} \delta_{\gamma_1,(l,s)}^{(n+1)}, \tag{24}$$

where

$$\zeta_{\gamma_1,1}^{(n+1)} = \left|\mathbf{I} - \mu_n(\mathbf{F}_1^{(n)})^H \overline{\mathbf{T}}_{\gamma_1}^{(n)} \mathbf{F}_1^{(n)}\right|, \tag{25}$$

$$\zeta_{\gamma_1,2}^{(n+1)} = \left|\mathbf{I} - \tau_n \mathbf{G}_{\omega_t^{(n)}}^{*} \widehat{\mathbf{T}}_{\gamma_1}^{(n)} \mathbf{G}_{\omega_t^{(n)}}^{T}\right|, \tag{26}$$

$$\overline{\mathbf{T}}_{\gamma_1}^{(n)} = \left(\mathbf{I}_{N_s} + \alpha^{(n+1)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n)}\widetilde{\mathbf{F}}_1^{(n)}\right)^{-1}, \tag{27}$$

$$\widehat{\mathbf{T}}_{\gamma_1}^{(n)} = \left(\mathbf{I}_{N_s} + \alpha^{(n)}\widetilde{\mathbf{G}}^{(n)} + \beta_1^{(n)}\widetilde{\mathbf{F}}_1^{(n)}\right)^{-1}, \tag{28}$$

and $\mu^{(n)} = \beta_1^{(n)} - \beta_1^{(n+1)}$.

By this time, we have successfully obtained the iterative equations of $\mathbf{\Psi}_1^{(n+1)} = \mathbf{\Psi}_1^{(n)}\zeta_{\psi_1,1}^{(n+1)}\delta_{\psi_1,s}^{(n+1)}$ and $\mathbf{\Gamma}_1^{(n+1)} = \mathbf{\Gamma}_1^{(n)}\zeta_{\gamma_1,2}^{(n+1)}\zeta_{\gamma_1,1}^{(n+1)}\delta_{\gamma_1,(l,s)}^{(n+1)}$. Similarly, we can continue in this fashion obtaining $\mathbf{\Psi}_2^{(n+1)} = \mathbf{\Psi}_2^{(n)}\zeta_{\psi_2,1}^{(n+1)}\delta_{\psi_2,s}^{(n+1)}$ and $\mathbf{\Gamma}_2^{(n+1)} = \mathbf{\Gamma}_2^{(n)}\zeta_{\gamma_2,2}^{(n+1)}\zeta_{\gamma_2,1}^{(n+1)}\delta_{\gamma_2,(l,s)}^{(n+1)}$. By substituting all the iterative equations to Eq. (12), we are able to arrive at our conclusion presented in Eq. (8). ■

***Remark** 1:* Theorem 1 serves to decouple the effect of AS in AF MIMO TWRCs when a holistic power model is considered. More specifically, the term $D^{(n+1)}$ represents the impact that the circuit power consumption has on the EE. The term $\Delta_{(l,s)}^{(n+1)}$ depicts the contribution to the EE when adding the pair of the lth receive and the sth transmit antenna at the $(n+1)$th iteration. Therefore, it guides us to select a pair of receive and transmit antennas that brings the largest EE contribution at each iteration. Moreover, since the term $P^{(n+1)}$ is a constant under given transmission power, the AS at each iteration can be equivalent to the following problem

$$(r(n+1), t(n+1)) = \underset{(l,s)\in\omega_r^c\times\omega_t^c}{\arg\max} \frac{\delta_{\gamma_1,(l,s)}^{(n+1)}\delta_{\gamma_2,(l,s)}^{(n+1)}}{\delta_{\psi_1,s}^{(n+1)}\delta_{\psi_2,s}^{(n+1)}}. \tag{29}$$

Therefore, according to Eq. (29), we add the $r(n+1)$th receive antenna and the $t(n+1)$th transmit antenna into the subsets $\omega_r^{(n)}$ and $\omega_t^{(n)}$ respectively at the $(n+1)$th iteration.

B. Power Adaptation and the Proposed Algorithm

In the subsection III-A, we gave the criterion for AS at each iteration under given transmission power. Now, we turn to investigate the power adaptation in order to maximize the EE for each iteration. Specifically, following the AS at each iteration, we calculate the optimal transmission power at the sources and relay that maximizes the EE for the current iteration. It can be proved that the numerator of EE is a concave function and the denominator is a convex function over $(P_{s,1}, P_{s,2}, P_r)$ under the constraints of our problem.[4] Thus the EE maximization problem for each iteration is pseudo-concave. As shown in Proposition 1, this pseudo-concave problem can be efficiently solved by using the tool of fractional programming which is related to a parametric program as stated in [16].

***Proposition** 1:* An optimization problem $\max\{\frac{R(x)}{P(x)}|x \in S\}$ is pseudo-concave when $R(x)$ is concave and $P(x)$ is convex for all $x \in S$. It can be related to the following parametric program

$$\max\{R(x) - qP(x)|x \in S\},\ q \in R. \tag{30}$$

[4]In fact, the numerator of EE is concave over $(P_{s,1}, P_{s,2}, P_r)$ except for very low SNR scenario, in which the minimum rate requirement would not be satisfied. The detailed proof is omitted due to space limitation.

Algorithm 1: Energy Efficient Relay AS Algorithm

Step 1: Initialization of the parameters and $L = 1$.

Step 2: Select one pair of antennas at the relay (l^*, s^*) according to Eq (29).

Step 3: Update the selected subsets ω_r^* and ω_t^* and calculate the optimal transmission power $(P_{s,1}^*, P_{s,2}^*, P_r^*)$ and maximum EE using the Dinkelbach method.

Step 4: Set the optimal transmission power tuple $(P_{s,1}^*, P_{s,2}^*, P_r^*)$ as the initial one for the next AS.

Step 5: Set $L = L+1$, if $L \leq N_r$, go to step 2; else stop.

Step 6: Choose the L with the largest EE and the corresponding antenna subsets and transmission power.

The maximum value can be achieved $q^* = \frac{R(x^*)}{P(x^*)}$ if and only if q^* and x^* satisfy $F(q^*) = F(q^*, x^*) = \max\{R(x) - qP(x)|x \in S\} = 0$. The root of the $F(q)$ can be efficiently found by the Dinkelbach method [16].

Based on Proposition 1, the Dinkelbach method is exploited to calculate the optimal transmission power tuple $(P_{s,1}^*, P_{s,2}^*, P_r^*)$ that maximizes the EE for each iteration. Then, this optimal power tuple is set as the initial transmission power tuple $(P_{s,1}, P_{s,2}, P_r)$ for the next iteration. In the following iterations, we can always update $(P_{s,1}, P_{s,2}, P_r)$ by the optimal one $(P_{s,1}^*, P_{s,2}^*, P_r^*)$ of the last iteration. It's worth pointing out that the AS for the first iteration is independent with the transmission power tuple because $\mathbf{T}_{\psi_k}^{(n)}$ and $\mathbf{T}_{\gamma_k}^{(n)}$, $k \in \{1,2\}$ are all identity matrices. Therefore, for the first iteration, the optimal pair of relay antennas can be selected without any initial transmission power.

According to Theorem 1 and the above-mentioned power adaptation process, an iterative energy efficient relay AS algorithm is proposed.[5] The details of the proposed algorithm are shown in Algorithm 1.

C. Analysis of the Computational Complexity

In this subsection, we turn to compare the computational complexities of our proposed algorithm and exhaustive search. It is worth pointing out that in both algorithms the same power adaption method is adopted to find the optimal transmission power for given antenna subsets. From [16], we know that the power adaption of each iteration can be done with a pleasant superlinear convergence. Therefore, the difference of the computational complexity between the two algorithms lies in the AS process. In particular, for each number of active relay RF chains L, exhaustive search has $\binom{N_r}{L}$ candidates for both the receive and transmit antenna subsets. Meanwhile, there is a pairing process between these candidates, which leads the candidates to $\binom{N_r}{L}^2 L!$ for each value of L. Therefore, the number of power adaptations in exhaustive search is $\sum_{L=1}^{N_r} \binom{N_r}{L}^2 L!$, which is exponential with respect to N_r. In contrast, our proposed algorithm needs only N_r power adaptations. In addition, the complexity of the AS process in our proposed algorithm is only $\mathcal{O}(N_s^2 N_r^3)$. Thus, the proposed algorithm significantly reduces the computational complexity.

[5]It is the fact that a slight fluctuation in the transmission power has little influence on the optimality of antenna selection makes our proposed algorithm feasible, which has been validated in our previous works [13], [17].

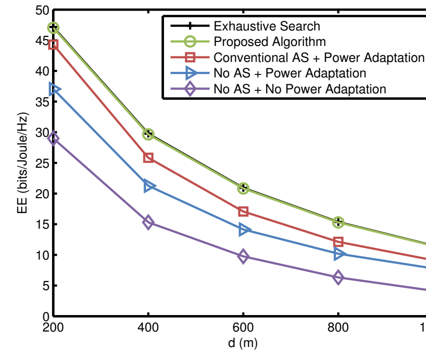

Fig. 2. Energy efficiency VS. the distance d with $\varepsilon = 0.5$

IV. Simulation Results

In this section, we provide simulation results to demonstrate the potential of the proposed algorithm. The EE is averaged over 2000 channel realizations. The values for $P_{c,S}$, $P_{c,R}$ and P_{c0} are 150mW, 100mW and 30mW, respectively [15]. $P_s^{\max} = P_r^{\max} = 300$ mW, $\eta_s = 0.38$, $\eta_r = 0.5$, $R_{\min} = 0.5$ (bits/s/Hz). $\varepsilon = d_{\mathrm{s_1 r}}/d$ is the ratio of the distance between $\mathcal{S}_1$ and $\mathcal{R}$, $d_{\mathrm{s_1 r}}$, and the distance between $\mathcal{S}_1$ and $\mathcal{S}_2$, d. The exponent of the log-distance path loss model $m = 4$. Thus, the average powers for $\mathbf{H}$ and $\mathbf{G}$ are given by $\sigma_H^2 = d_{\mathrm{s_1 r}}^{-m}$ and $\sigma_G^2 = (d - d_{\mathrm{s_1 r}})^{-m}$. The noise factors among the nodes are the same and equal to $N_0 = -174$ dBm/Hz. $N_s = N_r = 4$.

Figure 2 demonstrates the EE of different schemes as a function of the distance d between $\mathcal{S}_1$ and $\mathcal{S}_2$ for $\varepsilon = 0.5$. It can be seen that near-optimal performance is achieved by the proposed algorithm for all the distances when compared with exhaustive search. More specifically, among all the cases of 2000 channel realizations and different values of d, there is a percentage of 91.2% that the proposed algorithm achieves exactly the same EE as that obtained by exhaustive search. As to the number of active RF chains, there is a percentage of 93.6% that the proposed algorithm has the same number as that in exhaustive search. Moreover, we can see that the EE gain over the scheme composed of conventional AS with power adaptation is remarkable. This result confirms that conventional capacity-oriented AS schemes in which the number of active RF chains is fixed rather than dynamical cannot guarantee to be global energy efficient, while our energy efficient AS is capable of significantly improving EE when a holistic power model is considered. In addition, as we have pointed out, the power adaptation method in our proposed algorithm itself could be adopted to optimize EE in AF MIMO TWRCs without AS. This is also validated by our simulation result that the scheme with power adaption outperforms the scheme without power adaptation in terms of EE when AS

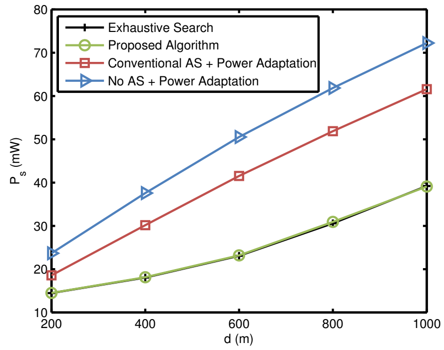

Fig. 3. The optimal transmission power VS. the distance d with $\varepsilon = 0.5$

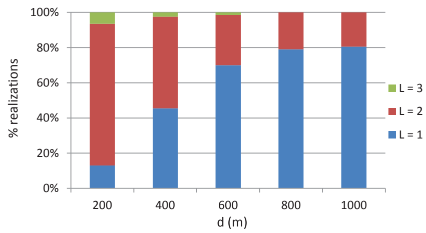

Fig. 4. Statistical results for the optimal number of active RF chains L VS. the distance d with $\varepsilon = 0.5$ when achieving the maximum EE

is not employed. Therefore, based on all the results, we can conclude the improvement of EE in our proposed algorithm benefits from both the energy efficient AS and the power adaption.

Figure 3 illustrates the total transmission power consumption of different schemes when achieving respective optimum EE varies with the exchange distance between $\mathcal{S}_1$ and $\mathcal{S}_2$, i.e., $P_s = P_{s,1}^* + P_{s,2}^*$. Besides the notable EE improvement, we can see that our proposed algorithm consumes much less transmission power. Moreover, the power advantage over other schemes increases significantly upon increasing of the exchange distance. This fact shows that our proposed algorithm are able to improve the EE without the cost of more transmission power; rather, our proposed energy efficient AS which achieves higher EE can also be alternatively exploited to reduce the transmission power in AF MIMO TWRCs.

Figure 4 shows the percentages for different numbers of active RF chains L among all the channel realizations when achieving the maximum EE with respect to the distance d. We can see that in short distances there exist cases that the optimal number of active RF chains equals three, i.e., $L = 3$, while in large distances L only lies between one and two when achieving the maximum EE. Moreover, $L = 2$ dominates in the short distances while $L = 1$ is the most possible case for large distances when achieving the optimal EE. Fig. 4 verifies that a dynamic change in the number of active antennas is of significant importance when the circuit power consumption is included in the power model.

V. Conclusion

We investigated the EE maximization with relay AS in AF MIMO TWRCs. An iterative energy efficient AS algorithm was proposed to jointly select the best relay antenna subsets, as well as optimize the transmission power at the two sources and relay. The joint iteration of active antennas and transmission power constitutes the core of the proposed algorithm. Simulation results demonstrated that the proposed algorithm enjoys a low complexity and achieves near-optimal performance. Moreover, it is capable of improving the EE, whilst reducing transmission power consumption.

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