How Many Antennas Should Be Activated in Keyhole Channels Under a Holistic Power Model

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***Abstract*—In this letter, we investigate the problem of how many transmit or receive antennas should be activated in keyhole channels with the consideration of a holistic power model. In conventional works where the circuit power is ignored, the system performance in terms of both ergodic capacity and outage probability improves with the increase of active antennas. However, when a holistic power model which includes the circuit power of RF chains as well as the transmission power is considered, there will exist a fundamental trade-off between the number of active antennas and the residual transmission power. Based on this trade-off, we derive a closed-form formula for the optimal number of active transmit or receive antennas in keyhole channels. In particular, by adopting the Meijer's G-function, we present concise expressions for the optimal numbers of active antennas that maximize the ergodic capacity and minimize the outage probability. Simulations will be shown in agreement with the theoretical results in a large extent.**

***Index Terms*—Keyhole channel, antenna, circuit power, ergodic capacity, outage probability.**

I. Introduction

MIMO (multiple-input-multiple-output) is an emerging technology that promises a significant increase in data rate and reliability for wireless communication systems [1]. It has been shown that the use of multiple transmit and receive antennas increases the information-theoretic capacity far beyond that of single-antenna systems in rich scattering propagation environment. However, in realistic propagation environment, the performance of MIMO could be severely degraded due to the *pinhole* or *keyhole* effects, which engender a rank-deficient channel and the loss of spatial degree of freedom [2]–[4]. In this case, it is more important and necessary in keyhole channels than in other MIMO channels to just use the "best" antennas rather than the whole multiple antennas.

Antenna selection (AS) in which only a subset of available antennas are activated for transmission and reception has been extensively investigated for MIMO systems under the condition of an idealistic rich scattering environment [5],[6]. The authors in [7] studied the AS in MIMO under keyhole condition and proved that AS in keyhole MIMO channels preserves the available diversity of the channel. In conventional works, the circuit power consumption of RF chains is not considered and the constraint is only the transmission power. The objective is just to select an optimal subset of antennas to maximize the channel capacity or minimize the bit-error rate (BER). However, a holistic power model which considers the circuit power consumption of RF chains as well as the transmission power is becoming imperative in that energy efficient communication has drawn more and more attention with the explosive increase in the energy consumption in communication systems [8]. In conventional works, systems achieve better performance with more active antennas. However, in the presence of the holistic power model, AS is significantly different [9], [10]. In this case, the number of active antennas has also to be optimized, rather than a fixed and given number, which proposes new challenges and demands. To the best of our knowledge, the AS for MIMO systems with circuit power consumption under the keyhole condition has not been investigated in a systematic way.

Motivated by this background, we shall study the optimization of antenna numbers in keyhole channels. We mark that there exists a fundamental trade-off. In his seminal paper [1], Telatar showed that the capacity of MIMO channels scales linearly with the minimum of the number of transmit and receive antennas, which indicates that MIMO channels have better performance with more antennas. However, when a holistic power model is considered, more power is depleted in the RF circuitry as the antenna numbers increase, therefore, the channel behavior is impaired. To achieve the best performance, we should balance the gains harvested from more antennas and the loss resulted from more power wastage. Hence, the antenna numbers should be determined accurately in this case.

In this letter, we shall present two results under the uncorrelated keyhole condition. First, we consider the maximization of ergodic capacity. As one of the most important information-theoretic measure, ergodic capacity has been discussed by many former researchers [1], [2]. We shall optimize the numbers of transmit and receive antennas for maximum ergodic capacity. Second, we optimize the number of receive antennas for minimum information outage probability. Outage probability is a more suitable measure in case of non-ergodic channels, which represents the probability that a certain information rate cannot be supported by an instantaneous realization of the channel [11], [12]. Our solution to optimal antenna numbers can be viewed as a dividing line, below which the performance improvements dominate when antenna numbers increase, while above which the circuitry wastage dominates. In this sense, hardware simplification techniques such as AS can be meaningful only below this line.

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II. System Model and Problem Formulation

A. System Model

We consider a point-to-point keyhole MIMO channel with t transmit and r receive antennas, which are statistically independent equal power components each with a circularly symmetric complex Gaussian distribution. The system model is given in [1] as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}. \quad (1)$$

Here $\mathbf{H} = \boldsymbol{\beta}\boldsymbol{\alpha}^T$ can be treated as a cascade of a multiple-input-single-output (MISO) channel and a single-input-multiple-output (SIMO) channel, in which $\boldsymbol{\alpha} \sim CN_t(0, \mathbf{I_t})$ and $\boldsymbol{\beta} \sim CN_r(0, \mathbf{I_r})$ denote the SIMO and MISO channel, respectively. As shown in [2], the channel capacity can be written in a scalar form as

$$C = \log_2\left(1 + \frac{P_t}{tN_0}XY\right), \quad (2)$$

where P_t denotes the transmission power; N_0 is the variance of the noise; $X = \|\alpha\|^2$ and $Y = \|\beta\|^2$ are central chi-square distributed with $2t$ and $2r$ degrees of freedom.

In this letter, we use the holistic power model in [8], where the transmission power and the circuit power constitute the total power P as

$$P = \frac{1}{\eta_{pa}}P_t + P_{c0} + tP_{ct} + rP_{cr}. \quad (3)$$

Here P_{ct} and P_{cr} are the power consumed by each transmit and receive RF chains; P_{c0} stands for the power of frequency synthesizer and other units of circuits; $\eta_{pa} < 1$ is the drain efficiency of the power amplifier.

B. Problem Formulation

In this letter, we focus on the optimization of two characteristics, namely, ergodic capacity maximization and outage probability minimization.

The first optimization objective is for transmit and receive antenna numbers to maximize ergodic capacity. It can be formulated as an integer programming problem given by

$$\max_{(t,r)} \langle C\rangle_{t,r}$$
$$st. \begin{cases} \frac{1}{\eta_{pa}}P_t + P_{c0} + tP_{ct} + rP_{cr} = P, \\ t, r \in \mathbb{N}^+. \end{cases} \quad (4)$$

Here ergodic capacity $\langle C\rangle_{t,r} = \mathbb{E}[C]$ is the mathematical expectation over the channel capacity; $\mathbb{N}^+$ denotes positive integers.

The second optimization objective is for receive antenna number to minimize the outage probability. It is formulated as an integer programming problem given by

$$\min_{r} p_{\text{out}}(t, r)$$
$$st. \begin{cases} \frac{1}{\eta_{pa}}P_t + P_{c0} + tP_{ct} + rP_{cr} = P, \\ t, r \in \mathbb{N}^+. \end{cases} \quad (5)$$

Here outage probability $p_{\text{out}}(t, r) = Pr[C < R]$ is the probability of that the channel is in outage, in which R denotes the outage capacity.

III. Optimization of Ergodic Capacity

In this section, we shall solve the optimization problem (4). The closed-form expression for ergodic capacity of spatially uncorrelated keyhole channels has been derived in [2] as

$$\langle C\rangle_{t,r} = \log_2\left(\frac{\rho}{t}\right) + \log_2(e)(\psi(t) + \psi(r)) + \frac{\log_2(e)}{\Gamma(t)\Gamma(r)} G^{3,2}_{2,4}\left(\frac{t}{\rho}\Big|_{r,t,1,0}^{1,1}\right), \quad (6)$$

where $\rho = \frac{P_t}{N_0}$ is the received SNR.

Since the optimization of (6) for positive integers (t, r) is difficult in general cases, we shall find the solution under a natural approximation shown in Theorem 1.

Theorem 1: Under the high transmission power condition, i.e, $P_t \gg P_{ct}, P_{cr}, P_{c0}, N_0$, the approximate optimal numbers of transmit and receive antennas are given by

$$\begin{cases} t^* \simeq \sqrt{\frac{P - P_{c0}}{4P_{ct}}}, \\ r^* \simeq \frac{P - P_{c0} - t^* P_{ct}}{2P_{cr}}. \end{cases} \quad (7)$$

Proof: First, we note that the received SNR is high under the high transmission power condition, i.e., $\rho \gg 1$, therefore, the term of Meijer's G-function vanishes. Substituting (3) into (6), we only need to maximize

$$\langle C\rangle_{t,r} \simeq \log_2\left(\frac{\eta_{pa}}{tN_0}(P - P_{c0} - tP_{ct} - rP_{cr})\right) + \log_2(e)\left(\psi(t) + \psi(r)\right). \quad (8)$$

Furthermore, the circuit power is low compared to transmission power under our assumption. Considering that the optimal t, r achieves infinity with negligible circuit power, we have $t, r \gg 1$ under the high "transmission to circuit power ratio" condition. Hence, (t, r) can be treated as continuous variables, and the optimal values are obtained by derivatives.

$$\frac{\partial}{\partial t}\langle C\rangle_{t,r} \simeq \log_2(e)\left(\psi'(t) - \frac{1}{t} - \frac{P_{ct}}{P - P_{c0} - tP_{ct} - rP_{cr}}\right),$$
$$\frac{\partial}{\partial r}\langle C\rangle_{t,r} \simeq \log_2(e)\left(\psi'(r) - \frac{P_{cr}}{P - P_{c0} - tP_{ct} - rP_{cr}}\right). \quad (9)$$

Using the asymptotic expansion

$$\psi(x) = \ln(x) - \frac{1}{2x} + \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}}, \quad (10)$$

where B_k represents the kth Bernoulli number, we can obtain the asymptotic optimal values (t^*, r^*) shown in (7) from $\frac{\partial}{\partial t}\langle C\rangle_{t,r}|_{(t^*,r^*)} = 0$ and $\frac{\partial}{\partial r}\langle C\rangle_{t,r}|_{(t^*,r^*)} = 0$.

In addition, the Hessian matrix is calculated as

$$\nabla^2\langle C\rangle_{t,r} = \log_2(e)\begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (11)$$

in which

$$A = \psi''(t) + \frac{1}{t^2} - \frac{P_{ct}^2}{(P - P_{c0} - tP_{ct} - rP_{cr})^2},$$
$$B = -\frac{P_{ct}P_{cr}}{(P - P_{c0} - tP_{ct} - rP_{cr})^2},$$
$$C = -\frac{P_{ct}P_{cr}}{(P - P_{c0} - tP_{ct} - rP_{cr})^2},$$
$$D = \psi''(r) - \frac{P_{cr}^2}{(P - P_{c0} - tP_{ct} - rP_{cr})^2}.$$

It can be checked that $A < 0$, $D < 0$, and $AD - BC > 0$, i.e, Hessian matrix is negative definite at (t^*, r^*), indicating that (7) finds the maximum point. ■

Remark 1: Compared with our result, in conventional situations where circuit power is neglected, the optimal $t^*, r^* \to \infty$, i.e., more antennas result into larger ergodic capacity.

IV. Optimization of Outage Probability

In this section, we shall solve the optimization problem (5). An analytical closed-form expression for outage probability has been derived by [13] as

$$p_{\text{out}}(t, r) = \frac{G_{1,3}^{2,1}(a|_{r,t,0}^{1})}{\Gamma(r)\Gamma(t)}, \qquad (12)$$

where $a = \frac{tN_0(2^R - 1)}{\eta_{pa}(P - P_{c0} - tP_{ct} - rP_{cr})}$ is a function of t and r.

Due to technical difficulties about the Meijer's G-function, it is hard to solve problem (5) directly. We shall constrain our discussion under the "high transmission power" condition, which is reasonable for actual systems, as well as results into a concise approximate solution, as shown in Theorem 2.

Theorem 2: Under the high transmission power condition, i.e, $P_t \gg P_{ct}, P_{cr}, P_{c0}$, the approximate optimal number of receive antennas is given by

$$r^* \simeq \frac{P - P_{c0} - tP_{ct}}{2P_{cr}}. \qquad (13)$$

Proof: Similar to the previous section, we can substitute (3) into (12) and take derivatives to seek the optimal r. We write $p_{\text{out}}(t, r) = f(a, t, r)$ as a composite function in which a is a function of t and r, and use chain rules

$$\frac{\partial}{\partial r} p_{\text{out}}(t, r) = \frac{\partial f(a, t, r)}{\partial a}\frac{\partial a}{\partial r} + \frac{\partial f(a, t, r)}{\partial r}. \qquad (14)$$

The first term regarding the partial derivative to a is

$$\frac{\partial f(a, t, r)}{\partial a}\frac{\partial a}{\partial r} = \frac{G_{1,3}^{2,1}(a|_{r,t,0}^{0})}{\Gamma(r)\Gamma(t)}\frac{\partial a}{a\partial r}. \qquad (15)$$

The second term regarding the partial derivative of r can be approximated by a discrete differential

$$\frac{\partial f(a, t, r)}{\partial r} \simeq f(a, t, r + 1) - f(a, t, r). \qquad (16)$$

By equation [14] (eq. 9.31.3) and equating different expressions for the first-order derivatives of Meijer's G-function, we can derive a recurrence relation as

$$rG_{1,3}^{2,1}(x|_{r,t,0}^{1}) = G_{1,3}^{2,1}(x|_{r,t,0}^{0}) + G_{1,3}^{2,1}(x|_{r+1,t,1}^{1}). \qquad (17)$$

TABLE I
SIMULATION PARAMETERS

Circuit power per transmit RF chain	$P_{ct} = 120$ mW
Circuit power per receive RF chain	$P_{cr} = 85$ mW
Circuit power in other units	$P_{c0} = 30$ mW
Drain efficiency	$\eta_{pa} = 0.35$
Equivalent noise	$N_0 = 64\ \mu$W

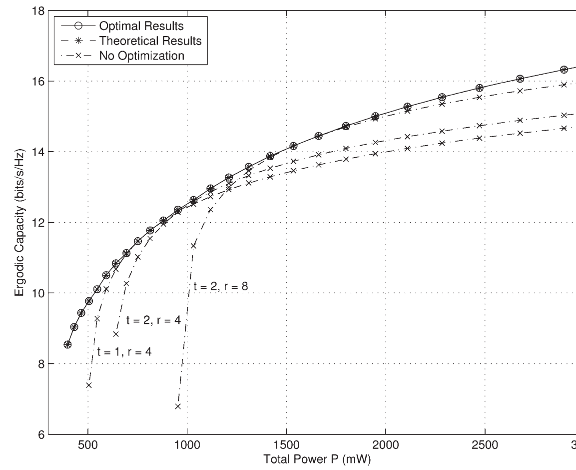

Fig. 1. Ergodic capacity VS. total power.

and the second term can be evaluated as

$$f(a, t, r + 1) - f(a, t, r) = -\frac{G_{1,3}^{2,1}(a|_{r,t,0}^{0})}{\Gamma(r)\Gamma(t)}\frac{1}{r}. \qquad (18)$$

In summary, we obtain an asymptotic expression as

$$\frac{\partial}{\partial r} p_{\text{out}}(t, r) \simeq \frac{G_{1,3}^{2,1}(a|_{r,t,0}^{0})}{\Gamma(r)\Gamma(t)}\left(\frac{\partial a}{a\partial r} - \frac{1}{r}\right), \qquad (19)$$

and (13) can be obtained from $\frac{\partial}{\partial r} p_{\text{out}}(t, r)|_{r^*} = 0$.

In addition, we can check that $\frac{\partial}{\partial r} p_{\text{out}}(t, r) < 0$ for $r < r^*$, while $\frac{\partial}{\partial r} p_{\text{out}}(t, r) > 0$ for $r > r^*$. It confirms that the r^* in (13) is the minimum point. ■

Remark 2: In conventional situations where circuit power is neglected, (18) demonstrates that $p_{\text{out}}(t, r + 1) < p_{\text{out}}(t, r)$, namely, the outage probability keeps decreasing as receive antenna number increases, resulting that the optimal $r^* \to \infty$.

Remark 3: The optimal number of receive antennas is actually the same for the two goals: when allocating a small amount of power in transmit RF chains, and nearly a half in receive chains ($r^* P_{ct} = P_t$), we can maximize the ergodic capacity as well as minimize the outage probability.

V. Numerical Results

In this section, we shall provide several numerical results for optimal antenna numbers in keyhole channels. Simulation parameters listed in Table I are adopted from [8].

We first show the optimal ergodic capacity for different total power consumption in Fig. 1. For each total power P, we seek the maximum ergodic capacity at exact optimal antenna numbers (Optimal Results), and calculate that at the approximate solution (t^*, r^*) shown in Theorem 1 (Theoretical Results). We also depict the ergodic capacity for three sets of fixed antenna numbers $(t, r) = (1, 2), (2, 4)$, and $(2, 8)$ as comparisons (No

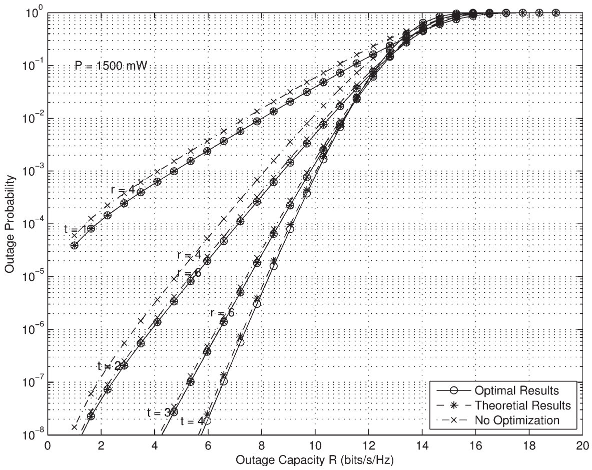

Fig. 2. Outage capacity distributions.

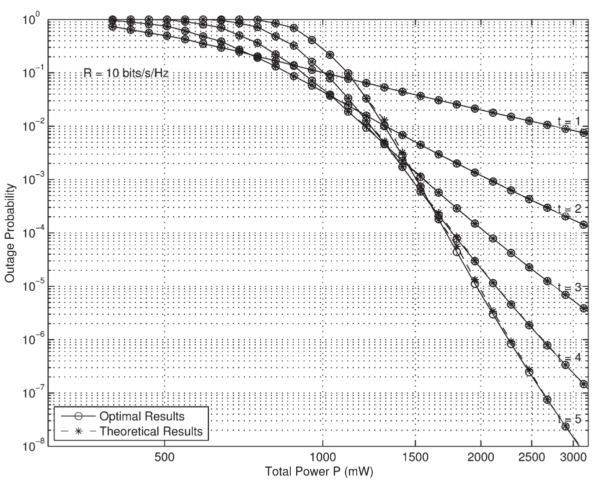

Fig. 3. Outage probability VS. total power.

Optimization). In this run, the exact solutions are totally the same with our approximate solutions (rounded to integers).

Next, considering that the outage probability is related to four variables t, r, R and P, we first set total power P fixed as 1500 mW and illustrate the outage capacity distributions in Fig. 2. For each outage capacity R and transmit antenna numbers t, we seek the minimum outage probability at the actual optimal receive antenna numbers (Optimal Results), and calculate that at the approximate solution r^* in Theorem 2 (Theoretical Results), and depict that for fixed antenna numbers $(t, r) = (1, 4)(2, 4), (2, 6), \text{and}(3, 6)$, as comparisons (No Optimization). It is shown that theoretical results have a near-optimal performance in regimes of high transmission to circuit power ratio.

Finally, we choose a fixed $R = 10$ bits/s/Hz and depict the relationship between outage probability and total power consumption in Fig. 3. For each total power P and transmit antenna number t, we seek the minimum outage probability at the actual optimal receive antenna numbers (Optimal Results), and calculate that at the approximate solution r^* in Theorem 2 (Theoretical Results). Theoretical results meet the actual optimal solutions in most cases, in which exceptions only exist in regimes where t is large and P is low, thus transmission power shrinks. It confirms our statement that Theorem 2 is a good approximation under the high transmission power condition. In addition, we mention that the intersections for different t on Fig. 3 imply that we can adjust numbers of active transmit antennas when power consumption changes.

VI. Conclusion

In this paper, we studied the ergodic capacity and information outage probability in keyhole channels under a holistic power model, and derived concise, closed-form solutions to optimal numbers of active antennas. First, we optimized numbers of active transmit and receive antennas for maximum ergodic capacity, which served as a good approximation to the optimal result. Second, by deriving an analytical closed-form expression in terms of Meijer's G-function, we optimized numbers of active receive antennas for minimum outage probability within the same model, which achieved the near-optimal performance under the high transmission power condition.

Since hardware simplification is extremely imperative for transmission through keyhole channels, our solutions could serve as the best numbers of antennas to balance the system performance and the energy efficiency. For other technologies such as antenna selection in keyhole channels, the active antenna numbers should not exceed that given by our solutions.

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