Greedy Relay Antenna Selection for Sum Rate Maximization in Amplify-and-Forward MIMO Two-Way Relay Channels Under a Holistic Power Model

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***Abstract*—This letter investigates the sum rate maximization in amplify-and-forward (AF) MIMO two-way relay channels (TWRCs) combined with relay antenna selection (AS). We consider the circuit power consumption in this letter and hence the optimization of the number of active RF chains is necessary. In particular, a greedy relay AS algorithm is proposed to avoid the exhaustive search. The proposed algorithm relies on a derived equation for the sum rate of AF MIMO TWRCs when relay AS is adopted, which guides us to select a pair of receive and transmit antennas that brings the largest sum rate increment at each selection. Simulation results show that the greedy algorithm is capable of achieving nearly the same performance of exhaustive search but with significantly reduced computational complexity.**

***Index Terms*—Antenna selection, amplify-and-forward, MIMO two-way relay channel, sum rate, holistic power model.**

I. Introduction

Two-way relaying constitutes an appealing spectral efficient transmission protocol as it is capable of compensating the spectral efficiency loss in one-way relaying. [1]. Meanwhile, the amplify-and-forward (AF) protocol has been widely used in relaying systems because of its low implementation complexity. Moreover, the performance of AF two-way relay channel (TWRC) can be further improved by incorporating the multiple-input multiple-output (MIMO) technique [2]. However, the multiple RF chains associated with multiple antennas in AF MIMO TWRCs will lead to extra energy cost when the circuit power consumption is considered [3]. Therefore, it is of paramount importance to design energy efficient AF MIMO TWRCs under a holistic power model.

Antenna selection (AS) is an attractive approach to avoid the aforementioned drawbacks by activating only a subset of available antennas for transmission and reception. AS for one-way AF MIMO relay systems was studied in [4]. Park *et al.* investigated AS in two-way MIMO relay systems and proposed computationally efficient AS algorithms to maximize the sum rate in the pioneer work [5]. In all these previous works, the circuit power consumption is not taken into account and the goal is that of finding optimal antenna subsets under the constraint of transmission power. However, the seminal work [6] has shown that AS under a holistic power model is considerably different from the conventional one in point-to-point MIMO systems. In this case, the number of active RF chains is also an important optimization parameter, which has remarkable influence on the performance.

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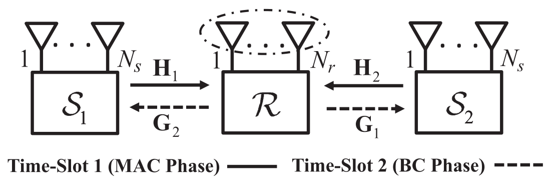

Fig. 1. System model of AF MIMO TWRC with relay AS.

In this letter, we extend the innovative results in [5] to the specific scenario where a holistic power model is considered. In this case, a dynamical adjustment of the number of relay antennas is necessary for the sum rate maximization. Therefore, our task is to find the optimal number of active RF chains at the relay as well as the corresponding receive and transmit relay antenna subsets. This problem is little similar to the subcarrier matching in multiple carrier relay channel, for which the optimal scheme is the *ordered subcarrier matching* [7]. This scheme is equivalent to the norm-based algorithm (NBS) in AS, which has been shown to only achieve good performance in low SNR regimes in [8]. For the sake of circumventing the prohibitive computational complexity in exhaustive search and attaining a good performance in different cases, we propose a greedy relay antenna subset selection algorithm, which selects a pair of receive and transmit antennas to maximize the sum rate increment at each iteration. More specifically, the criterion for the AS at each iteration is based on the derived equation for the sum rate in AF MIMO TWRCs with relay AS. Simulation results show that the proposed algorithm is capable of achieving near-optimal performance while at lower computational complexities.

II. System Model and Problem Formulation

As illustrated in Fig. 1, we consider an AF MIMO TWRC consisting of two source nodes ($\mathcal{S}_1$ and $\mathcal{S}_2$), and one relay node ($\mathcal{R}$), where $\mathcal{S}_1$ and $\mathcal{S}_2$ are both equipped with N_s antennas and $\mathcal{R}$ has N_r antennas. We assume that due to heavy path-loss, there is no direct link between the source nodes. $\mathbf{H}_1$ and $\mathbf{H}_2$ denote the channel matrices from $\mathcal{S}_1$ and $\mathcal{S}_2$ to $\mathcal{R}$, respectively. Channel reciprocity is herein assumed to hold for TWRC; thus, the channels from $\mathcal{R}$ to $\mathcal{S}_2$ and $\mathcal{S}_1$ are given by $\mathbf{G}_1 = \mathbf{H}_2^T$

and $\mathbf{G}_2 = \mathbf{H}_1^T$, respectively. Assume that all the channels are Rayleigh flat fading with average power varied with the path loss. Suppose the selected subsets of active receive and transmit antennas at $\mathcal{R}$ are respectively denoted by ω_r and ω_t, then the $|\omega_r| \times N_s$ subchannel matrices for the first time-slot (MAC phase) are given by $\mathbf{H}_1^{\omega_r}$ and $\mathbf{H}_2^{\omega_r}$. Similarly, the $N_s \times |\omega_t|$ subchannels for the second time-slot (BC phase) are denoted by $\mathbf{G}_1^{\omega_t}$ and $\mathbf{G}_2^{\omega_t}$, respectively. Since each RF chain at the relay actually connects one receive antenna and one transmit antenna, once we activate one more RF chain at the relay, we actually activate a pair of receive and transmit antennas. Therefore, we have $L = |\omega_r| = |\omega_t|$, which is the number of active RF chains at $\mathcal{R}$. Furthermore, all the nodes are assumed to operate in half-duplex mode, and hence the message exchange between the two source nodes accomplishes in two time-slots.

In the first time-slot of AF TWRC, $\mathcal{S}_1$ and $\mathcal{S}_2$ simultaneously transmit their own signals to $\mathcal{R}$ through the selected subchannels. During the second time-slot, $\mathcal{R}$ processes the received signal with AF relay operation, and then broadcasts the processed signal to source nodes via the active transmit antennas. With the adoption of analogue network coding (ANC) and perfect channel state information (CSI), the achievable rate from $\mathcal{S}_1$ to $\mathcal{S}_2$, and the achievable rate from $\mathcal{S}_2$ to $\mathcal{S}_1$ with relay AS are respectively given by

$$R_1 = \frac{1}{2}\log_2 \frac{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_1^{\omega_t}\left(\mathbf{G}_1^{\omega_t}\right)^H + \alpha\rho_1 \mathbf{F}_1\mathbf{F}_1^H\right|}{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_1^{\omega_t}\left(\mathbf{G}_1^{\omega_t}\right)^H\right|}, \tag{1}$$

$$R_2 = \frac{1}{2}\log_2 \frac{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_2^{\omega_t}\left(\mathbf{G}_2^{\omega_t}\right)^H + \alpha\rho_2 \mathbf{F}_2\mathbf{F}_2^H\right|}{\left|\mathbf{I}_{N_s} + \alpha \mathbf{G}_2^{\omega_t}\left(\mathbf{G}_2^{\omega_t}\right)^H\right|}, \tag{2}$$

where $\mathbf{F}_1 = \mathbf{G}_1^{\omega_t}\mathbf{H}_1^{\omega_r}$ and $\mathbf{F}_2 = \mathbf{G}_2^{\omega_t}\mathbf{H}_2^{\omega_r}$. $\rho_1 = P_{s,1}/(N_s N_0)$ and $\rho_2 = P_{s,2}/(N_s N_0)$, where $P_{s,1}$ and $P_{s,2}$ are the transmission power at $\mathcal{S}_1$ and $\mathcal{S}_2$, and N_0 is the additive noise power. $\alpha = \frac{P_r/N_0}{\rho_1 \mathrm{tr}(\mathbf{H}_1^{\omega_r}\mathbf{H}_1^{\omega_r H}) + \rho_2 \mathrm{tr}(\mathbf{H}_2^{\omega_r}\mathbf{H}_2^{\omega_r H}) + L}$ is the power coefficient satisfying the relay power constraint P_r. Therefore, the achievable sum rate of AF MIMO TWRCs with relay AS can be given by

$$R_{\mathrm{sum}} = R_1 + R_2. \tag{3}$$

In this letter, the holistic power which considers the circuit power consumption consists of three parts. In particular, P_1 and P_2 respectively denote the overall power consumption between $\mathcal{S}_1$ and $\mathcal{R}$, and $\mathcal{S}_2$ and $\mathcal{R}$ during the first time-slot. P_3 denotes the overall power consumption in the second time-slot.

$$P_1 = \frac{1}{\eta_s}\cdot P_{s,1} + N_s \cdot P_{ct,S} + \frac{1}{2}|\omega_r|\cdot P_{cr,R}, \tag{4}$$

$$P_2 = \frac{1}{\eta_s}\cdot P_{s,2} + N_s \cdot P_{ct,S} + \frac{1}{2}|\omega_r|\cdot P_{cr,R}, \tag{5}$$

$$P_3 = \frac{1}{\eta_r}\cdot P_r + 2N_s \cdot P_{cr,S} + |\omega_t|\cdot P_{ct,R}, \tag{6}$$

where $P_{ct,S}$ and $P_{ct,R}$ denote the power consumed by each source and relay RF chain for transmission. $P_{cr,S}$ and $P_{cr,R}$ are the power consumption of each source and relay RF chain for reception. η_s and η_r are the drain efficiency of the power amplifier at the source and relay, respectively.

The main objective of this letter is to select the best active antenna subsets ω_r^* and ω_t^* at $\mathcal{R}$ that maximize the sum rate under the constraints of $L = |\omega_r| = |\omega_t|$ and the given maximum allowable power $P_k^{\max}$, $k \in \{1, 2, 3\}$. Mathematically, our objective can be represented by

$$\max_{\omega_r \subset \mathcal{N}, \omega_t \subset \mathcal{N}} R_{\mathrm{sum}}$$
$$s.t. \begin{cases} 1 \le |\omega_r| = |\omega_t| \le N_{\max} \\ 0 < P_k \le P_k^{\max}, \ k \in \{1,2,3\} \end{cases} \tag{7}$$

where $\mathcal{N} = \{1, 2, \ldots, N_r\}$. $N_{\max}$ is the maximum allowable number of data streams under any given power tuple (P_1, P_2, P_3), which is given by

$$N_{\max} = \min\{N_{\max,r,1}, N_{\max,r,2}, N_{\max,t}, N_s\}, \tag{8}$$

where $N_{\max,r,i} = \lfloor 2(P_i - N_s \cdot P_{ct,S})/P_{cr,R}\rfloor$, $i \in \{1, 2\}$ and $N_{\max,t} = \lfloor (P_3 - 2N_s \cdot P_{cr,S})/P_{ct,R}\rfloor$.

III. Antenna Selection Algorithms

In this section, we first propose a greedy relay antenna selection algorithm for the optimization problem defined in Eq. (7) in order to avoid the prohibitive complexity of exhaustive search. Then, we present the computational complexity analysis of different algorithms.

A. The Greedy Antenna Selection Algorithm

The core idea of the proposed algorithm is that we judiciously select a pair of receive and transmit antennas that maximizes the sum rate at each iteration. The selection criterion is based on Theorem 1, which gives the equation of the sum rate when a pair of relay receive and transmit antenna is selected at each iteration. For clarity, the selected active receive and transmit antenna subsets after n iterations are respectively denoted by $\omega_{r,n} = \{r(1), r(2), \ldots, r(n)\}$ and $\omega_{t,n} = \{t(1), t(2), \ldots, t(n)\}$, where $r(k)$ and $t(k)$ stand for the indices for the kth selected receive and the kth selected transmit antenna, respectively. Meanwhile, the corresponding subchannels $\mathbf{H}_\tau^{\omega_{r,n}}$ and $\mathbf{G}_\tau^{\omega_{t,n}}$ are denoted by $\mathbf{H}_{\tau,n}$ and $\mathbf{G}_{\tau,n}$, $\tau \in \{1, 2\}$, i.e., the number of active receive and transmit antennas are both n. The achievable rates for the nth iteration are given by $R_{\tau,n}$ and $R_{\mathrm{sum},n}$, respectively. At the $(n+1)$th iteration, if the l^*th receive and the s^*th transmit antenna of the relay are selected and added into $\omega_{r,n}$ and $\omega_{t,n}$, i.e., $r(n+1) = l^*$, $t(n+1) = s^*$, the new subchannel matrices are denoted by $\mathbf{H}_{\tau,n+1}$, $\mathbf{G}_{\tau,n+1}$.

We first present the main results in the letter as follows. With the selection of the receive and transmit antenna pair (l, s) at the relay, the achievable sum rate of AF MIMO TWRC under a holistic power constraint is updated by the following equation[1]

$$R_{\mathrm{sum},n+1} = R_{\mathrm{sum},n} + \Upsilon_{n+1} + \Delta_{(l,s),n+1}, \tag{9}$$

where

$$\Upsilon_{n+1} = \frac{1}{2}\log_2 \frac{\prod_{\tau=1}^{2} \vartheta_{\theta_\tau,n+1}\zeta_{\theta_\tau,n+1}}{\prod_{\tau=1}^{2} \varsigma_{\phi_\tau,n+1}} \tag{10}$$

[1]Due to space limitation, $\delta_{\phi_\tau,s,n+1}$, $\varsigma_{\phi_\tau,n+1}$, $\delta_{\theta_\tau,(l,s),n+1}$, $\vartheta_{\theta_\tau,n+1}$, $\zeta_{\theta_\tau,n+1}$ are given in Eqs. (17), (21), (23), (25), and (26), respectively.

$$\Delta_{(l,s),n+1} = \frac{1}{2}\log_2 \frac{\prod_{\tau=1}^{2} \delta_{\theta_\tau,(l,s),n+1}}{\prod_{\tau=1}^{2} \delta_{\phi_\tau,s,n+1}}. \quad (11)$$

The derivations for the main results go as follows. For convenience, let $\mathbf{\Phi}_{\tau,n} = |\mathbf{I}_{N_s} + \alpha_n \mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H|$ and $\mathbf{\Theta}_{\tau,n} = |\mathbf{I}_{N_s} + \alpha_n \mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \alpha_n \rho_{\tau,n}\mathbf{F}_{\tau,n}\mathbf{F}_{\tau,n}^H|$, $\tau \in \{1,2\}$ be the denominators and numerators of Eqs. (1) and (2), respectively. In this context, $\rho_{\tau,n} = P_{s,\tau,n}/(N_s N_0)$ and $\alpha_n = \frac{P_{r,n}/N_0}{\rho_{1,n}\mathrm{tr}(\mathbf{H}_{1,n}\mathbf{H}_{1,n}^H)+\rho_{2,n}\mathrm{tr}(\mathbf{H}_{2,n}\mathbf{H}_{2,n}^H)+n}$. On account of the holistic power model, $P_{s,\tau,n} = \eta_s\big(P_\tau - N_s P_{ct,S} - \frac{1}{2}nP_{cr,R}\big)$ and $P_{r,n} = \eta_r(P_3 - 2N_s P_{cr,S} - nP_{ct,R})$. Therefore, the achievable sum rate for the $(n+1)$th iteration is given by

$$R_{\mathrm{sum},n+1} = \sum_{\tau=1}^{2} \frac{1}{2}\log_2 \frac{\mathbf{\Theta}_{\tau,n+1}}{\mathbf{\Phi}_{\tau,n+1}}. \quad (12)$$

From Eq. (12), we know that the key problem lies in finding the respective sequential equations of $\mathbf{\Phi}_{\tau,n+1}$ and $\mathbf{\Theta}_{\tau,n+1}$.

We start with deriving the sequential equation of $\mathbf{\Phi}_{\tau,n+1} = \left|\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n+1}\mathbf{G}_{\tau,n+1}^H\right|$. Suppose the sth antenna is selected and added into the active transmit subsets, then we have the following matrix update

$$\mathbf{G}_{\tau,n+1}\mathbf{G}_{\tau,n+1}^H = \mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \mathbf{g}_{\tau,s}\mathbf{g}_{\tau,s}^H, \quad (13)$$

where $\mathbf{g}_{\tau,s}$ is the sth column of $\mathbf{G}_\tau$. Substituting Eq. (13) into $\mathbf{\Phi}_{\tau,n+1}$, we have

$$\mathbf{\Phi}_{\tau,n+1} = \left|\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \alpha_{n+1}\mathbf{g}_{\tau,s}\mathbf{g}_{\tau,s}^H\right|. \quad (14)$$

By invoking the matrix determinant lemma [9], that is

$$|\mathbf{A} + \mathbf{u}\mathbf{v}^H| = |\mathbf{A}|(1 + \mathbf{v}^H\mathbf{A}^{-1}\mathbf{u}), \quad (15)$$

where $\mathbf{A}$ is an invertible square matrix and $\mathbf{u}$, $\mathbf{v}$ are column vectors, we can simplify Eq. (14) to

$$\mathbf{\Phi}_{\tau,n+1} = \left|\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H\right| \delta_{\phi_\tau,s,n+1}, \quad (16)$$

where

$$\delta_{\phi_\tau,s,n+1} = 1 + \alpha_{n+1}\mathbf{g}_{\tau,s}^H \mathbf{T}_{\phi_\tau,n+1}\mathbf{g}_{\tau,s}, \quad (17)$$

$$\mathbf{T}_{\phi_\tau,n+1} = \left(\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H\right)^{-1}. \quad (18)$$

Since the first term of Eq. (16) is still not the exact $\mathbf{\Phi}_{\tau,n} = \left|\mathbf{I}_{N_s} + \alpha_n\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H\right|$, further simplification is needed. By rewriting $\alpha_{n+1} = \alpha_n - \lambda_n$ in Eq. (16) and adopting the generalization of matrix determinant lemma [9] to it, i.e.,

$$|\mathbf{A} + \mathbf{U}\mathbf{V}^H| = |\mathbf{A}||\mathbf{I} + \mathbf{V}^H\mathbf{A}^{-1}\mathbf{U}|, \quad (19)$$

we can obtain the sequential equation for $\mathbf{\Phi}_{\tau,n+1}$

$$\begin{aligned}\mathbf{\Phi}_{\tau,n+1} &= \left|\mathbf{I}_{N_s} + \alpha_n\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H - \lambda_n\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H\right| \delta_{\phi_\tau,s,n+1}\\ &= \mathbf{\Phi}_{\tau,n}\varsigma_{\phi_\tau,n+1}\delta_{\phi_\tau,s,n+1}, \quad (20)\end{aligned}$$

where

$$\varsigma_{\phi_\tau,n+1} = \left|\mathbf{I} - \lambda_n\mathbf{G}_{\tau,n}^H\left(\mathbf{I}_{N_s} + \alpha_n\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H\right)^{-1}\mathbf{G}_{\tau,n}\right|. \quad (21)$$

We are left with the task of finding the sequential equation for $\mathbf{\Theta}_{\tau,n+1}$. With the matrix update and the rank-2 update of determinant [5], [9], i.e.,

$$|\mathbf{A} + \mathbf{u}\mathbf{v}^H + \mathbf{v}\mathbf{u}^H| = |\mathbf{A}|\left(|t|^2 - ab\right), \quad (22)$$

where $t = 1 + \mathbf{v}^H\mathbf{A}^{-1}\mathbf{u}$, $a = \mathbf{u}^H\mathbf{A}^{-1}\mathbf{u}$, $b = \mathbf{v}^H\mathbf{A}^{-1}\mathbf{v}$ and $\mathbf{A}$ is a Hermitian matrix, $\mathbf{\Theta}_{\tau,n+1}$ could be simplified as $\mathbf{\Theta}_{\tau,n+1} = |\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \beta_{\tau,n+1}\mathbf{F}_{\tau,n}\mathbf{F}_{\tau,n}^H|\delta_{\theta_\tau,(l,s),n+1}$ and $\delta_{\theta_\tau,(l,s),n+1}$ is given by Eq. (23)

$$\begin{aligned}\delta_{\theta_\tau,(l,s),n+1} &= \left(1 + \mathbf{q}^H\mathbf{T}_{\theta_\tau,n+1}\mathbf{g}_{\tau,s}\right)^2\\ &\quad - \left(\mathbf{g}_{\tau,s}^H\mathbf{T}_{\theta_\tau,n+1}\mathbf{g}_{\tau,s}\right)\left(\mathbf{q}^H\mathbf{T}_{\theta_\tau,n+1}\mathbf{q}\right). \quad (23)\end{aligned}$$

$\mathbf{h}_{\tau,l}$ is the transpose of the lth row of $\mathbf{H}_\tau$, $\beta_{\tau,n+1} = \alpha_{n+1}\rho_{\tau,n+1}$, $\mathbf{q} = \frac{\alpha_{n+1}+\beta_{\tau,n+1}\|\mathbf{h}_{\tau,l}\|^2}{2}\mathbf{g}_{\tau,s} + \beta_{\tau,n+1}\mathbf{F}_{\tau,n}\mathbf{h}_{\tau,l}^*$ and $\mathbf{T}_{\theta_\tau,n+1} = (\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \beta_{\tau,n+1}\mathbf{F}_{\tau,n}\mathbf{F}_{\tau,n}^H)^{-1}$. Substituting $\alpha_{n+1} = \alpha_n - \lambda_n$ and $\beta_{\tau,n+1} = \beta_{\tau,n} - \varepsilon_{\tau,n}$ into $\mathbf{\Theta}_{\tau,n+1}$ and using the generalization of matrix determinant lemma, we can obtain

$$\mathbf{\Theta}_{\tau,n+1} = \mathbf{\Theta}_{\tau,n}\vartheta_{\theta_\tau,n+1}\zeta_{\theta_\tau,n+1}\delta_{\theta_\tau,(l,s),n+1}, \quad (24)$$

where

$$\vartheta_{\theta_\tau,n+1} = \left|\mathbf{I} - \lambda_n\mathbf{G}_{\tau,n}^H\overline{\mathbf{T}}_{\theta_{\tau,n+1}}\mathbf{G}_{\tau,n}\right|, \quad (25)$$

$$\zeta_{\theta_\tau,n+1} = \left|\mathbf{I} - \varepsilon_{\tau,n}\mathbf{F}_{\tau,n}^H\widehat{\mathbf{T}}_{\theta_{\tau,n+1}}\mathbf{F}_{\tau,n}\right|, \quad (26)$$

$$\overline{\mathbf{T}}_{\theta_{\tau,n+1}} = \left(\mathbf{I}_{N_s} + \alpha_n\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \beta_{\tau,n}\mathbf{F}_{\tau,n}\mathbf{F}_{\tau,n}^H\right)^{-1}, \quad (27)$$

$$\widehat{\mathbf{T}}_{\theta_{\tau,n+1}} = \left(\mathbf{I}_{N_s} + \alpha_{n+1}\mathbf{G}_{\tau,n}\mathbf{G}_{\tau,n}^H + \beta_{\tau,n}\mathbf{F}_{\tau,n}\mathbf{F}_{\tau,n}^H\right)^{-1}. \quad (28)$$

At this time, we have successfully found the sequential equations of both $\mathbf{\Phi}_{\tau,n+1}$ and $\mathbf{\Theta}_{\tau,n+1}$. Therefore, the results can be obtained by substituting Eqs. (20) and (24) into Eq. (12).

Remark 1: Theorem 1 divides the update of the sum rate at each iteration into two parts. The term Υ_{n+1} is only related with the selection via the term α_{n+1}, which is dominated by the terms $\rho_{1,n+1}$ and $\rho_{2,n+1}$ in high SNR regimes and has little influence on the selection in low SNR regimes where the selection is degraded into norm-based scheme [8]. By ignoring Υ_{n+1} in Eq. (9), it therefore guides us to mainly focus on the maximization of the term $\Delta_{(l,s),n+1}$, which could be equivalent to the following problem

$$(r(n+1), t(n+1)) = \underset{(l,s)\in\omega_{r,n}^c\times\omega_{t,n}^c}{\arg\max} \frac{\prod_{\tau=1}^{2}\delta_{\theta_\tau,(l,s),n+1}}{\prod_{\tau=1}^{2}\delta_{\phi_\tau,s,n+1}}, \quad (29)$$

where $\omega_{r,n}^c = \mathcal{N} - \omega_{r,n}$ and $\omega_{t,n}^c = \mathcal{N} - \omega_{t,n}$. According to Eq. (29), we add the $r(n+1)$th receive antenna and the $t(n+1)$th transmit antenna into the subsets $\omega_{r,n}$ and $\omega_{t,n}$ respectively at the $(n+1)$th iteration.

Based on the discussion above, a computationally efficient relay AS algorithm is proposed to address the optimization problem defined in Eq. (7). Thanks to Eqs. (9) and (29) serving as the core of our proposed algorithm, we are able to judiciously select nearly the best antennas which bring the largest increment for each iteration. We also record the sum rate of each iteration at the same time. After $N_{\max}$ iterations, we choose the antenna subsets of the iteration which has the largest sum rate as the final results.

B. Analysis of the Computational Complexity

In the light of the complexity analysis in [5], [8], we can give the computational complexity for each algorithm. In particular,

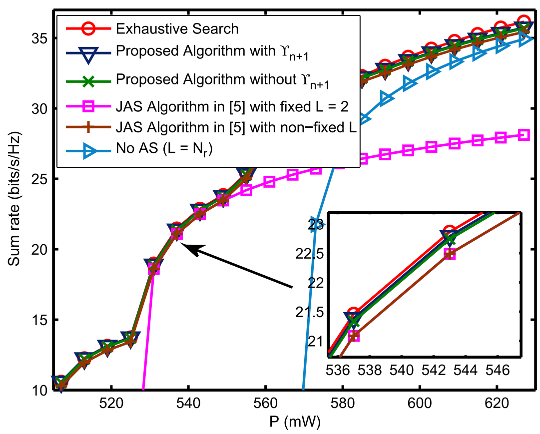

Fig. 2. Sum rate as a function of P with $\varepsilon = 0.5$, $d = 400$ m.

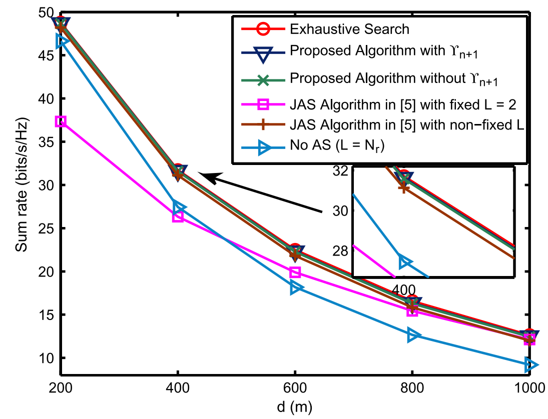

Fig. 3. Sum rate as a function of d with $\varepsilon = 0.5$, $P = 580$ mW.

exhaustive search needs go over all the antenna subsets combinations with respect to different numbers of active RF chains at $\mathcal{R}$. Thus, the overall possible combinations of exhaustive search are $\sum_{L=1}^{N_{\max}} \binom{N_r}{L}^2 L!$, which is nearly exponential with regard to N_r. Moreover, we consider the JAS algorithm in [5] with the extension of non-fixed L in our simulations, the complexity of which is given by $\mathcal{O}(N_s^2 N_r^2 N_{\max}^2)$. Without the repeated use of rank-1 update at each step in JAS algorithm, our greedy algorithm makes forward a step and enjoys a complexity of $\mathcal{O}(N_s^2 N_r^2 N_{\max})$.

IV. Simulation Results

In this section, we provide simulation results to demonstrate the potential of the proposed algorithm. The results are averaged over 2000 channel realizations. Simulation parameters are as follows [3]. $P_{ct,S}$, $P_{cr,S}$, $P_{ct,R}$, $P_{cr,R}$ are 120 mW, 85 mW, 50 mW, and 45 mW, respectively. $\eta_s = 0.38$, $\eta_r = 0.5$, $B = 10$ MHz. The noise factors among the nodes are the same and equal to $N_0 = -174$ dBm/Hz. $\varepsilon = d_{\mathrm{s_1 r}}/d$ is the ratio of the distance between $\mathcal{S}_1$ and $\mathcal{R}$, $d_{\mathrm{s_1 r}}$, and the distance between $\mathcal{S}_1$ and $\mathcal{S}_2$, d. The exponent of the log-distance path loss model is $m = 4$. Thus, the average powers for $\mathbf{H}_1$ and $\mathbf{H}_2$ are given by $\sigma_{H_1}^2 \propto d_{\mathrm{s_1 r}}^{-m}$ and $\sigma_{H_2}^2 \propto (d - d_{\mathrm{s_1 r}})^{-m}$. $N_s = N_r = 4$.

Fig. 2 shows the sum rate versus total power $P_1^{\max} = P_2^{\max} = P$, $P_3^{\max} = 2P$ at the distance $d = 400$ m for $\varepsilon = 0.5$. It can be seen that the proposed greedy algorithm achieves nearly the same sum rate as compared to exhaustive search for the entire total power regime (as expected, Υ_{n+1} has little influence on the performance). In contrast, the JAS algorithm with fixed L can only approach to the optimal sum rate within a narrow range of total power. Moreover, the performance of our proposed algorithm is still slightly better than that of the JAS algorithm combined with non-fixed L.

Fig. 3 demonstrates the sum rate versus the distance d with $\varepsilon = 0.5$ and $P_3^{\max} = 2P_1^{\max} = 2P_2^{\max} = 2P = 1160$ mW. It can be seen that the proposed algorithm achieves near-optimal performance over all the distances. More importantly, Fig. 3 gives us a better understanding of the key trade-off between the transmission power and the spatial multiplexing in MIMO TWRC with AS under a holistic power model. Specifically, in the cases when the sum rate is limited by the power, i.e., long distance transmission, less active relay antennas is preferred for sum rate maximization as more power could be allocated to transmit data, and hence a high received SNR is achieved. This is the reason for the fact that the curve of JAS algorithm with fixed $L = 2$ is above that of No AS ($L = 4$) when d is more than 500 m. On the other hand, in the cases when the sum rate is limited by the spatial multiplexing, activating more relay antennas is preferred as it can increase the spatial degrees of freedom, which would bring a significant improvement for the sum rate. This explains the fact that the curve of No AS ($L = 4$) is above that of JAS algorithm with fixed $L = 2$ when d is less than 500 m.

V. Conclusion

A greedy relay AS algorithm has been proposed to maximize the sum rate in AF MIMO TWRCs where the circuit power consumption is considered. The proposed algorithm relies on the derived sum rate equation in this letter. It has been shown by simulation results that the proposed algorithm is capable of significantly reducing the computational complexity, whilst attaining near-optimal performance compared to exhaustive search.

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