Iterative Antenna Selection for Decode-and-Forward MIMO Relay Systems Under a Holistic Power Model

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***Abstract*—In this letter, we investigate the antenna selection in decode-and-forward (DF) MIMO relay systems where the circuit power consumption is considered. The main problem is that the optimal active relay antenna subsets can only be selected by exhaustive search. To reduce the complexity, three iterative properties on the capacity bounds are first derived in this letter, which lay the foundation for our proposed algorithm. This algorithm can improve the performance of DF MIMO relay systems by the maximization of the upper and lower bounds on the capacity. Simulation results will show that the proposed low complexity algorithm has nearly the same performance as that of exhaustive search. In addition, it has a remarkable performance gain over the conventional DF MIMO relay protocol.**

***Index Terms*—MIMO relay, antenna selection, iterative, decode-and-forward, holistic power model.**

I. INTRODUCTION

WIRELESS relaying combined with the use of multiple antennas has become a key-factor in modern communications for its benefits of a large diversity gain and a large spectrum efficiency. The upper and lower bounds on the capacity for full-duplex MIMO relay systems are derived in [1]. In practice, however, adopting multiple antennas will consume more circuit power when a holistic power model is considered [2]. Therefore, it's necessary and important to design high performance MIMO relay systems under a holistic power model.

Antenna selection in which only a subset of available antennas are active for transmitting or receiving is a good choice to reduce the power consumption without incurring much performance loss. Many attentions have been paid to the antenna selection in one-hop MIMO systems [3]–[5]. In [6], a joint discrete stochastic algorithm which combines transmit diversity selection and relay selection is proposed for DF MIMO relay systems. Fast antenna subset selection algorithms for two-hop half-duplex MIMO relay systems without regard to the direct link between the source and destination can be found in [7], [8]. In conventional works, the circuit power consumption of RF chains is not considered and the constraint is only the transmission power. The goal is just to select an optimal subset of antennas or relays to maximize the channel capacity or minimize the bit-error rate (BER). However, the pioneer works [9], [10] have shown that the antenna selection under a holistic power model is significantly different from conventional works for one-hop MIMO systems. In this case, the number of active antennas has also to be optimized, which proposes new challenges and demands on signal processing units. To the best of our knowledge, the antenna selection for MIMO relay systems with circuit power consumption has not been studied in a systematic way.

Motivated by this background, we propose an iterative and near-optimal relay antenna selection algorithm for DF MIMO relay systems where the circuit power is considered. Our goal is to select the best active receive and transmit antenna subsets at the relay that maximize the upper or lower bounds on the capacity. This iterative algorithm is based on the observation that there exists iterative properties of capacity bounds with antenna selection. Therefore, we select one pair of receive and transmit antennas which lead to the highest increment of achievable rate at each step. Simulation results show that the proposed low complexity algorithm has nearly the same performance as that of exhaustive search.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a DF MIMO relay communication system which consists of three nodes. The source, relay and destination are equipped with N_s, N_r and N_d antennas, respectively. The channels between the source and the destination, the source and the relay, and the relay and the destination are denoted by $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively. Assume that all the channels are Rayleigh flat fading with average power varied with the path loss. The noise vectors at the relay and destination are AWGN (Additive White Gaussian Noise) with power σ_r^2 and σ_d^2, respectively. Moreover, the relay node has two sets of antennas, one for reception and the other for transmission. Thus, it operates in the full-duplex mode. According to [1], the upper bound and lower bound on the capacity are given by

$$C \leq C_{\text{upper}} = \min\left\{C_1(\mathbf{H}_1), C_2(\mathbf{H}_2)\right\}, \tag{1}$$

where $C_1(\mathbf{H}_1) = \log\det(\mathbf{I}_{N_s} + \gamma_1\mathbf{H}_1^H\mathbf{H}_1 + \gamma_0\mathbf{H}_0^H\mathbf{H}_0)$, $C_2(\mathbf{H}_2) = \log\det(\mathbf{I}_{N_d} + \gamma_0\mathbf{H}_0\mathbf{H}_0^H + \gamma_2\mathbf{H}_2\mathbf{H}_2^H)$, and

$$C \geq C_{\text{lower}} = \max\left\{C_d, \min\left\{C_3(\mathbf{H}_1), C_2(\mathbf{H}_2)\right\}\right\}, \tag{2}$$

where $C_d = \log\det(\mathbf{I}_{N_d} + \gamma_0\mathbf{H}_0\mathbf{H}_0^H)$, $C_3(\mathbf{H}_1) = \log\det(\mathbf{I}_{N_r} + \gamma_1\mathbf{H}_1\mathbf{H}_1^H)$. Note that $\gamma_0 = \frac{P_s}{\sigma_d^2 N_s}$, $\gamma_1 = \frac{P_s}{\sigma_r^2 N_s}$ and $\gamma_2 = \frac{P_r}{\sigma_d^2 N_r}$. P_s and P_r are the transmission power of the source and relay, respectively.

In this letter, the holistic power model whose main feature is the inclusion of the power consumption of RF chains can be divided into the following two parts:

$$P_1 = \frac{1}{\eta_{pa}} \cdot P_s + N_s \cdot P_{ct} + |\omega_r| \cdot P_{cr,\mathrm{R}}, \tag{3}$$

$$P_2 = \frac{1}{\eta_{pa}} \cdot P_r + N_d \cdot P_{cr} + |\omega_t| \cdot P_{ct,\mathrm{R}}, \tag{4}$$

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where ω_r and ω_t are the selected receive and transmit relay antenna subsets. $L=|\omega_r|=|\omega_t|$ is the number of active RF chains at the relay. P_{ct} and $P_{ct,R}$ are the power consumed by each source and relay RF chain for transmission. P_{cr} and $P_{cr,R}$ are the power consumed by each destination and relay RF chain for reception. η_{pa} is the drain efficiency of the power amplifier [2].

Our aim is to find the optimal ω_r^* and ω_t^* that maximize the upper or lower bounds under the constraints of $|\omega_r|=|\omega_t|$ and the given maximum allowable power $P_1^{\max}$ and $P_2^{\max}$, which are expressed by the following optimization problems.

$$\max_{\tilde{\mathbf{H}}_1\in\boldsymbol{\mathcal{H}}_1,\tilde{\mathbf{H}}_2\in\boldsymbol{\mathcal{H}}_2} \min\left\{C_1(\tilde{\mathbf{H}}_1), C_2(\tilde{\mathbf{H}}_2)\right\}$$
$$s.t. \begin{cases} 1\le|\omega_r|=|\omega_t|\le N_{\max} \\ 0<P_1\le P_1^{\max},\ 0<P_2\le P_2^{\max} \end{cases} \tag{5}$$

and

$$\max_{\tilde{\mathbf{H}}_1\in\boldsymbol{\mathcal{H}}_1,\tilde{\mathbf{H}}_2\in\boldsymbol{\mathcal{H}}_2} \left\{C_d, \min\left\{C_3(\tilde{\mathbf{H}}_1), C_2(\tilde{\mathbf{H}}_2)\right\}\right\}$$
$$s.t. \begin{cases} 1\le|\omega_r|=|\omega_t|\le N_{\max} \\ 0<P_1\le P_1^{\max},\ 0<P_2\le P_2^{\max} \end{cases} \tag{6}$$

where $\boldsymbol{\mathcal{H}}_1$ and $\boldsymbol{\mathcal{H}}_2$ are the sets of all $|\omega_r|\times N_s$ and $N_d\times|\omega_t|$ possible subchannels of $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively. $N_{\max}$ is the maximum allowable number of active RF chains for a given pair P_1 and P_2, which is given by

$$N_{\max}=\min\{N_{\max,r},N_{\max,t}\}, \tag{7}$$

where $N_{\max,r}=\lfloor(P_1-N_s\cdot P_{ct})/P_{cr,R}\rfloor$ and $N_{\max,t}=\lfloor(P_2-N_d\cdot P_{cr})/P_{ct,R})\rfloor$.

III. Antenna Selection Algorithm

In this section, we first propose an iterative antenna selection algorithm for the optimization problems (5) and (6). Then, we analyze the computational complexities of different algorithms.

Exhaustive search is the only optimal method which is yet complexity prohibitive. Therefore, an iterative algorithm is proposed to select one pair of receive and transmit relay antennas that lead to the highest increment of the achievable rate at each iteration, which is based on Theorem 1. We denote the channel matrices after n steps of antenna selection by $\mathbf{H}_{1,n}$ and $\mathbf{H}_{2,n}$, i.e., the numbers of active receive and transmit antennas are both n. At the $(n+1)$th step, if the s_r^*th receive and s_t^*th transmit antenna of the relay are selected and then added into the active antenna subsets, the new $(n+1)\times N_s$ and $N_d\times(n+1)$ channel matrices are denoted by $\mathbf{H}_{1,n+1}$ and $\mathbf{H}_{2,n+1}$, respectively. $\mathbf{h}_s$ is a column vector which stands for the conjugate transpose of the sth row of $\mathbf{H}_1$ and $\mathbf{g}_s$ is the sth column of $\mathbf{H}_2$.

Theorem 1: With the antenna selection at the relay, C_1, C_2 and C_3 could be expressed by the following iterative equations when the holistic power model is considered

$$C_1(\mathbf{H}_{1,n+1})=C_1(\mathbf{H}_{1,n})+A_1(\mathbf{H}_{1,n})+B_1(\mathbf{H}_{1,n})+\Delta_{1,s,n}, \tag{8}$$

$$C_2(\mathbf{H}_{2,n+1})=C_2(\mathbf{H}_{2,n})+A_2(\mathbf{H}_{2,n})+B_2(\mathbf{H}_{2,n})+\Delta_{2,s,n}, \tag{9}$$

and $C_3(\mathbf{H}_{1,n+1})=C_3(\mathbf{H}_{1,n})+B_3(\mathbf{H}_{1,n})+\Delta_{3,s,n}$.

Proof: Please refer to the Appendix. ■

Remark 1: The notations used in Theorem 1 are defined as follows.

Notations for C_1:

$$A_1(\mathbf{H}_{1,n})=\log\det\left(\mathbf{I}_{N_d}-\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H\right), \tag{10}$$

$$B_1(\mathbf{H}_{1,n})=\log\det\left(\mathbf{I}_n-\frac{\eta_{pa}P_{cr,R}}{\sigma_r^2N_s}\mathbf{H}_{1,n}\left(\mathbf{I}_{N_s}+\gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0+\gamma_{1,n}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n}\right)^{-1}\mathbf{H}_{1,n}^H\right), \tag{11}$$

$$\Delta_{1,s,n}=\log\left(1+\gamma_{1,n+1}\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{h}_s\right)+\log\left(1+\alpha_1\mathbf{h}_1^H\left(\mathbf{I}_{N_d}-\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H\right)^{-1}\mathbf{h}_1\right), \tag{12}$$

$$\mathbf{T}_{1,n}=\left(\mathbf{I}_{N_s}+\gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0+\gamma_{1,n+1}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n}\right)^{-1}, \tag{13}$$
$$\mathbf{h}_1=\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{h}_s, \tag{14}$$
$$\alpha_1=\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\cdot\frac{\gamma_{1,n+1}}{1+\gamma_{1,n+1}\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{h}_s}. \tag{15}$$

It can be seen that after the derivation $\mathbf{h}_s$ only exists in the term $\Delta_{1,s,n}$, which is actually the essential part as to the selection of relay receive antenna for each step. Moreover, the circuit power also has influence on the receive antenna selection due to existence of $P_{cr,R}$ in $\Delta_{1,s,n}$.

Notations for C_2:

$$A_2(\mathbf{H}_{2,n})=\log\det\left(\mathbf{I}_{N_s}-\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\mathbf{H}_0^H\mathbf{T}_{2,n}\mathbf{H}_0\right), \tag{16}$$

$$B_2(\mathbf{H}_{2,n})=\log\det\left(\mathbf{I}_{N_d}-m_n\mathbf{H}_{2,n}^H\left(\mathbf{I}_n+\gamma_{0,n}\mathbf{H}_0\mathbf{H}_0^H+\gamma_{2,n}\mathbf{H}_{2,n}\mathbf{H}_{2,n}^H\right)^{-1}\mathbf{H}_{2,n}\right), \tag{17}$$

$$\Delta_{2,s,n}=\log\left(1+\gamma_{2,n+1}\mathbf{g}_s^H\mathbf{T}_{2,n}\mathbf{g}_s\right)+\log\left(1+\alpha_2\mathbf{g}_1^H\left(\mathbf{I}_{N_s}-\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\mathbf{H}_0^H\mathbf{T}_{2,n}\mathbf{H}_0\right)^{-1}\mathbf{g}_1\right), \tag{18}$$

$$\mathbf{T}_{2,n}=\left(\mathbf{I}_{N_d}+\gamma_{0,n}\mathbf{H}_0\mathbf{H}_0^H+\gamma_{2,n+1}\mathbf{H}_{2,n}\mathbf{H}_{2,n}^H\right)^{-1}, \tag{19}$$
$$\mathbf{g}_1=\mathbf{H}_0^H\mathbf{T}_{2,n}\mathbf{g}_s, \tag{20}$$
$$m_n=\gamma_{2,n}-\gamma_{2,n+1}, \tag{21}$$
$$\alpha_2=\frac{\eta_{pa}P_{cr,R}}{\sigma_d^2N_s}\cdot\frac{\gamma_{2,n+1}}{1+\gamma_{2,n+1}\mathbf{g}_s^H\mathbf{T}_{2,n}\mathbf{g}_s}. \tag{22}$$

It can be seen that after the derivation $\mathbf{g}_s$ only exists in the term $\Delta_{2,s,n}$, which is actually the essential part as to the selection of relay transmit antenna for each step. Also, the circuit power $P_{ct,R}$ in $\gamma_{2,n+1}$ and $P_{cr,R}$ both affect the selection of transmit antenna.

Notations for C_3:

$$B_3(\mathbf{H}_{1,n})=\log\left(\mathbf{I}_n-\frac{\eta_{pa}P_{cr,R}}{\sigma_r^2N_s}\mathbf{H}_{1,n}\left(\mathbf{I}_{N_s}+\gamma_{1,n}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n}\right)^{-1}\mathbf{H}_{1,n}^H\right), \tag{23}$$

$$\Delta_{3,s,n}=\log\left(1+\gamma_{1,n+1}\mathbf{h}_s^H\left(\mathbf{I}_{N_s}+\gamma_{1,n+1}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n}\right)^{-1}\mathbf{h}_s\right). \tag{24}$$

Algorithm 1 The Antenna Selection Algorithm

Step 1:Initialize all the channels, noise factors and power. $N_{\max}$ is computed based on (7), set $n = 1$.

Step 2:Select the pair of antennas at the relay (s_r^*, s_t^*) according to (25) and (26), respectively.

Step 3:Update channel matrices $\mathbf{H}_{1,n}$ and $\mathbf{H}_{2,n}$ as well as $\Delta_{1,s,n}$, $\Delta_{2,s,n}$ and $\Delta_{3,s,n}$. Compute $C_{\mathrm{upper},n}$ and $C_{\mathrm{lower},n}$ according to (1) and (2), respectively.

Step 4:Set $n = n + 1$, if $n \le N_{\max}$, go to step 2; else stop.

Step 5:Choose the n with the largest $C_{\mathrm{upper},n}$ or $C_{\mathrm{lower},n}$ and the corresponding antenna subsets as the results.

Remark 2: Theorem 1 helps to decouple the effect of antenna selection in DF MIMO relay systems. In particular, $A_\tau(\mathbf{H}_{\tau,n})$, $\tau = \{1, 2\}$ and $B_\upsilon(\mathbf{H}_{\upsilon,n})$, $\upsilon = \{1, 2, 3\}$ represent the effect of the circuit power consumption. The contribution of adding the sth receive or transmit antenna is captured by the term $\Delta_{\kappa,s,n}$, $\kappa = \{1, 2, 3\}$. Therefore, it motivates us to find the pair of receive and transmit antennas that bring the largest contribution at each step, which can be equivalent to the following problems

$$s_r^* = \begin{cases} \arg\max_s \Delta_{1,s,n}; & \text{upper bound maximization} \\ \arg\max_s \Delta_{3,s,n}; & \text{lower bound maximization} \end{cases} \quad (25)$$

$$s_t^* = \arg\max_s \Delta_{2,s,n}. \quad (26)$$

Based on Theorem 1 and Remark 2, an efficient iterative algorithm is proposed. The details of the algorithm are presented in Algorithm I. The selection for the receive antennas is captured by (25) according to different objectives, i.e., upper bound maximization or lower bound maximization. The selection for the transmit antennas is the same for both upper and lower bounds maximization, which is captured by (26). For each selection, there are at most N_r candidates in the proposed algorithm. Thus, the computation complexity of the worst case $(N_{\max} = N_r)$ is $\mathcal{O}(\max(N_s, N_d)N_r^2)$. In contrast, for each number of active RF chains L, exhaustive search has $\binom{N_r}{L}$ candidates for both the receive and transmit antenna selections. Thus, the overall combinations in exhaustive search are $\sum_{L=1}^{N_{\max}} \binom{N_r}{L}^2$. Consider the worst case $N_{\max} = N_r$, the computation complexity is exponential with respect to N_r. Therefore, the proposed algorithm significantly reduces the computation complexity.

IV. Simulation Results

In this section, we provide simulation results to demonstrate the potential of the proposed algorithm. The results are averaged over 2000 channel realizations. Parameters for the simulation are as follows. P_{ct}, P_{cr}, $P_{ct,R}$ and $P_{cr,R}$ are 120 mW, 85 mW, 50 mW and 45 mW, respectively. $B = 10$ MHz, $\eta_{pa} = 0.38$, $N_s = N_r = N_d = 8$. The noise factors among the nodes are the same and equal to $\sigma^2 = -174$ dBm/Hz. $\beta = d_{\mathrm{sr}}/d$ is the ratio of the distance between the source and relay d_{sr}, and the distance between the source and destination d. $n = 4$ is the path loss exponent. Therefore, the average powers for $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ are given by $\sigma_{\mathrm{sd}}^2 = d^{-n}$, $\sigma_{\mathrm{sr}}^2 = d_{\mathrm{sr}}^{-n}$ and $\sigma_{\mathrm{rd}}^2 = (d - d_{\mathrm{sr}})^{-n}$, respectively.

Fig. 1 shows the capacity bounds versus total power $P_1^{\max} = P_2^{\max} = P$ at the distance $d = 400$ m for $\beta = 0.5$. It is easy to find that near-optimal performance can be achieved by the proposed algorithm for different power consumption. In addition, it can be seen that the proposed algorithm outperforms

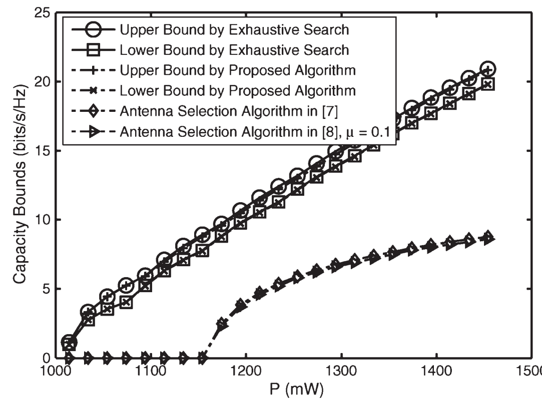

Fig. 1. Capacity bounds as a function of P with $\beta = 0.5$, $d = 400$ m.

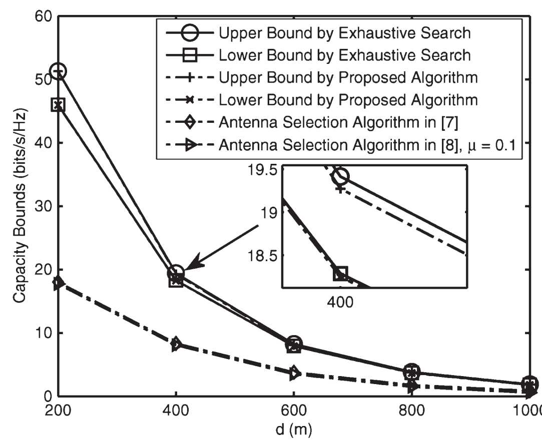

Fig. 2. Capacity bounds as a function of d with $\beta = 0.5$, $P = 1400$ mW.

the antenna selection algorithms proposed in [7], [8]. Among all the cases of 2000 channel realizations and different values of total power consumption P, there is a percentage of 90.2% that the proposed algorithm has the same number of antennas as that in exhaustive search to achieve near-optimal performance. Among the remaining 9.8% cases, there is a percentage of 5.3% that one more antenna is activated in the proposed algorithm. In the other 4.5% cases, one less antenna is active in the proposed algorithm. Based on these results, we may conclude that the proposed algorithm is capable of attaining a solution considering the number of selected antennas that is close to the number of antennas that the exhaustive search algorithm achieves.

Fig. 2 demonstrates the performance of different algorithms for capacity bounds as a function of d with $\beta = 0.5$ and $P_1^{\max} = P_2^{\max} = P = 1400$ mW. It can be seen that the proposed algorithm achieves nearly the same performance as that of exhaustive search over all the distances. The upper bound gets closer to the lower bound as d increases. Moreover, a remarkable capacity gain over the antenna selection algorithms proposed in [7], [8] can be observed.

V. Conclusion

An iterative antenna selection algorithm has been developed to improve the performance of DF MIMO relay systems by the maximization of the upper and lower bounds on the capacity. Three iterative properties which lay the foundation of the proposed algorithm have been first derived in this letter. It greatly reduces the computational complexity yet with little performance loss when compared to exhaustive search.

APPENDIX A
PROOF OF THEOREM 1

Here, we take the proof for C_1 as an example. The same method can be applied to the derivations of C_2 and C_3. For the $(n+1)$th step, the corresponding normalized SNRs are expressed by

$$\gamma_{0,n+1} = \frac{\eta_{pa}\left(P_1 - N_s \cdot P_{ct} - (n+1)P_{cr,R}\right)}{\sigma_d^2 N_s}, \quad (27)$$

$$\gamma_{1,n+1} = \frac{\eta_{pa}\left(P_1 - N_s \cdot P_{ct} - (n+1)P_{cr,R}\right)}{\sigma_r^2 N_s}. \quad (28)$$

Thus C_1 for the $(n+1)$th step is as follows

$$C_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_s} + \gamma_{1,n+1}\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} + \gamma_{0,n+1}\mathbf{H}_0^H\mathbf{H}_0\right). \quad (29)$$

Since $\gamma_{0,n+1} = \gamma_{0,n} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}$

$$C_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_s} + \gamma_{1,n+1}\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0^H\mathbf{H}_0\right). \quad (30)$$

By using the generalization of matrix determinant lemma, we obtain

$$C_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_s} + \gamma_{1,n+1}\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0\right) + A_1(\mathbf{H}_{1,n+1}), \quad (31)$$

where

$$A_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\left(\mathbf{I}_{N_s} + \gamma_{1,n+1}\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0\right)^{-1}\mathbf{H}_0^H\right). \quad (32)$$

Noting that $\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} = \mathbf{H}_{1,n}^H\mathbf{H}_{1,n} + \mathbf{h}_s\mathbf{h}_s^H$, and applying the matrix determinant lemma to the first summand of (31), we obtain that

$$C_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_s} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 + \gamma_{1,n+1}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n}\right) + \tilde{\Delta}_{1,s,n} + A_1(\mathbf{H}_{1,n+1}), \quad (33)$$

where $\tilde{\Delta}_{1,s,n} = \log(1 + \gamma_{1,n+1}\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{h}_s)$, $\mathbf{T}_{1,n} = (\mathbf{I}_{N_s} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 + \gamma_{1,n+1}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n})^{-1}$.

Based on the fact that $\gamma_{1,n+1} = \gamma_{1,n} - \frac{\eta_{pa}P_{cr,R}}{\sigma_r^2 N_s}$ and the generalization of matrix determinant lemma, (33) can be further reduced by

$$C_1(\mathbf{H}_{1,n+1}) = C_1(\mathbf{H}_{1,n}) + B_1(\mathbf{H}_{1,n}) + \tilde{\Delta}_{1,s,n} + A_1(\mathbf{H}_{1,n+1}), \quad (34)$$

where $B_1(\mathbf{H}_{1,n}) = \log\det(\mathbf{I}_n - \frac{\eta_{pa}P_{cr,R}}{\sigma_r^2 N_s}\mathbf{H}_{1,n}(\mathbf{I}_{N_s} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 + \gamma_{1,n}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n})^{-1}\mathbf{H}_{1,n}^H)$.

As $A_1(\mathbf{H}_{1,n+1})$ is a function of $\mathbf{H}_{1,n+1}$, we need to further reduce it. First, we rewrite it as follows

$$A_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\tilde{\mathbf{T}}_{1,n}\mathbf{H}_0^H\right), \quad (35)$$

where $\tilde{\mathbf{T}}_{1,n} = (\mathbf{I}_{N_s} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 + \gamma_{1,n+1}\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1})^{-1}$.

Since $\mathbf{H}_{1,n+1}^H\mathbf{H}_{1,n+1} = \mathbf{H}_{1,n}^H\mathbf{H}_{1,n} + \mathbf{h}_s\mathbf{h}_s^H$, $\tilde{\mathbf{T}}_{1,n}$ can be expressed by $\tilde{\mathbf{T}}_{1,n} = (\mathbf{I}_{N_s} + \gamma_{0,n}\mathbf{H}_0^H\mathbf{H}_0 + \gamma_{1,n+1}\mathbf{H}_{1,n}^H\mathbf{H}_{1,n} + \gamma_{1,n+1}\mathbf{h}_s\mathbf{h}_s^H)^{-1}$. Applying the Woodbury matrix formula to it yields

$$\tilde{\mathbf{T}}_{1,n} = \mathbf{T}_{1,n} - \frac{\gamma_{1,n+1}}{1 + \gamma_{1,n+1}\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{h}_s}\mathbf{T}_{1,n}\mathbf{h}_s\mathbf{h}_s^H\mathbf{T}_{1,n}. \quad (36)$$

Combining (35) and (36), we can obtain

$$A_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H + \alpha_1\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{h}_s\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{H}_0^H\right), \quad (37)$$

where $\alpha_1 = \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s} \cdot \frac{\gamma_{1,n+1}}{1+\gamma_{1,n+1}\mathbf{h}_s^H\mathbf{T}_{1,n}\mathbf{h}_s}$.

To make (37) more clear, we denote $\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{h}_s$ by $\mathbf{h}_1$. Then, it follows that

$$A_1(\mathbf{H}_{1,n+1}) = \log\det\left(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H + \alpha_1\mathbf{h}_1\mathbf{h}_1^H\right). \quad (38)$$

Once again, we apply the matrix determinant lemma to (38) and get

$$A_1(\mathbf{H}_{1,n+1}) = A_1(\mathbf{H}_{1,n}) + \log\left(1 + \alpha_1\mathbf{h}_1^H\left(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H\right)^{-1}\mathbf{h}_1\right), \quad (39)$$

where $A_1(\mathbf{H}_{1,n}) = \log\det(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H)$.

Finally, combining (34) and (39), we obtain the result as follows

$$C_1(\mathbf{H}_{1,n+1}) = C_1(\mathbf{H}_{1,n}) + A_1(\mathbf{H}_{1,n}) + B_1(\mathbf{H}_{1,n}) + \Delta_{1,s,n}, \quad (40)$$

where $\Delta_{1,s,n} = \tilde{\Delta}_{1,s,n} + \log(1 + \alpha_1\mathbf{h}_1^H(\mathbf{I}_{N_d} - \frac{\eta_{pa}P_{cr,R}}{\sigma_d^2 N_s}\mathbf{H}_0\mathbf{T}_{1,n}\mathbf{H}_0^H)^{-1}\mathbf{h}_1)$.

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